American Journal of Epidemiology

Vol. 00, No. 00
https://doi.org/10.1093/aje/kwab178
Advance Access publication:

## Practice of Epidemiology

# "Translating" All-Cause Mortality Rate Ratios or Hazard Ratios to Age-, Longevity-, and Probability-Based Measures

**Menglan Pang and James A. Hanley***

* Correspondence to Dr. James Hanley, Department of Epidemiology, Biostatistics and Occupational Health, Faculty of Medicine and Health Sciences, McGill University, 845 Sherbrooke Street West, Montreal, QC H3A 0G4, Canada (e-mail: james.hanley@mcgill.ca).

*Initially submitted April 20, 2020; accepted for publication June 10, 2021.*

Epidemiologists commonly use an adjusted hazard ratio or incidence density ratio, or a standardized mortality ratio, to measure a difference in all-cause mortality rates. They seldom translate it into an age-, time-, or probability-based measure that would be easier to communicate and to relate to. Several articles have shown how to translate from a standardized mortality ratio or hazard ratio to a longevity difference, a difference in actuarial ages, or a probability of being outlived. In this paper, we describe the settings where these translations are and are not appropriate and provide some of the heuristics behind the formulae. The tools that yield differences in "effective age" and in longevity are applicable when both 1) the mortality rate ratio (hazard ratio) is constant over age and 2) the rates themselves are log-linear in age. The "probability/odds of being outlived" metric is applicable whenever the first condition holds, and thus it provides no direct information on the magnitude of the effective age/longevity difference.

Gompertz' law; life expectancy; proportional hazards; standardized mortality ratio

Abbreviations: HR, hazard ratio; MLB, Major League Baseball; NFL, National Football League; RLE, remaining life expectancy; SMR, standardized mortality ratio.

> *Major League Baseball players tend to live about 24% longer than the average American man, according to a new study led by researchers from Harvard T.H. Chan School of Public Health* (1).

This statement recently appeared in an article in the *Harvard Gazette* (1). Suppose we were to take it literally and start the longevity contest at, say, age 45 years, so that the American man would live an average of a further 35 years, to about 80 years. According to the Harvard headline, the Major League Baseball (MLB) players would live to an average age of about 100, some 20 years longer than the average American man. But does this make sense, or is there a "translation error" somewhere?

When one goes back to the source publication (2), it does not take long to see where the numbers got lost in translation. The main result was that "compared with US males, the MLB players had significantly lower mortality rates from all causes (SMR, 0.76; 95% CI, 0.73–0.78)" (2, p. 1298). So the standardized mortality ratio (SMR)—or hazard ratio (HR)—of 0.76 was converted to a 24% lower mortality rate, and then, in the public relations item, to a 24% longer duration of life.

The reporting of another finding from this work—involving a direct comparison between the death rates of National Football League (NFL) players and MLB players—by a different news agency (3) was more accurate. Under the title "Former NFL Players Die at a Faster Rate Than Other Professional Athletes, Study Finds," we read that it found that NFL players "died at a rate that was almost 1.3 times higher" (3) than MLB players.

Three tools have been developed to translate these types of ratios. The first focuses on an age difference. To arrive at it, Spiegelhalter (4), influenced by Brenner et al. (5), used a law created by the actuary Benjamin Gompertz (6) to translate an HR into an "effective age": the age of a "healthy" person who has the same risk profile as the individual in question. He gave several examples, involving lifestyle, of this "useful and attractive metaphor" for "vividly communicat[ing] risks to individuals" (4). One example was an HR of 2.20 associated with smoking 20 cigarettes per day, a behavior which "adds" 8 years to one's chronological age—that is, it puts a 50-year-old smoker in the same risk category as a 58-year-old nonsmoker.

The second tool, yielding a life-expectancy difference, was developed by Haybittle (7). Based on earlier work (8), he used Gompertz' law to go from an SMR or HR of 0.76 or 1.30 to a longevity difference measured in years. His examples focused on translating changes in population mortality rates over calendar time. Era comparisons involving entire populations were also featured in the work of others (9, 10) who have refined Haybittle's formula.

The third tool translates the HR to a probability, which we term the "*probability of being outlived*." It exploits a relationship that was explicitly addressed in the 1950s (11, 12) and again immediately following Cox's 1972 article (13). The probability, which has been given various names in the subsequent decades (14), makes fewer assumptions about the mortality rates than the other two do. So far, examples of this probability have come from clinical medicine and have involved recovery times or undesired clinical events.

Unfortunately, none of these 3 tools appears to have had much traction in the reporting of results of chronic disease epidemiology studies. University media offices need help in translating SMRs and HRs into quantities that journalists, and the public at large, can more readily relate to; but to do so correctly, there first needs to be a broader awareness in the chronic disease epidemiology community of the available tools and when they are applicable. In this article, we aim to raise this awareness; it directly addresses readers of a journal that has for many decades emphasized chronic disease epidemiology and its associated statistical methods but has usually confined its effect measures to differences or ratios of risks and rates.

The remainder of this paper is structured as follows. We begin by specifying the age/time dimension over which 2 compared hazard functions are assumed to bear a constant ratio to each other (to be proportional) and consider when this constancy can be expected to apply. We then show historical and modern instances where each of the 2 hazard functions exhibits an additional feature, first noticed by Gompertz (6), and consider the contexts in which this additional feature can be expected to apply. We then 1) provide a brief heuristic explanation of the Gompertz-based age shift involved in Spiegelhalter's "effective age" formula; 2) derive a rough shrinkage factor that further translates the age shift in the mortality rate scale to a difference in remaining life expectancy (RLE) beyond a specified age—the quantity addressed by Haybittle; and 3) provide a less specific probability metric that, unlike the age-shift and life-expectancy metrics, does not depend on Gompertz' law. For didactic purposes, we show calculations to 2 decimal places throughout; in practice, any age shift or longevity difference or probability should be coarsened considerably.

Our primary target audience is epidemiologists and science writers who do not have access to the raw data but need to be aware of their options when translating the SMRs and HRs reported in a published article for a broader audience.

## PROPORTIONAL HAZARDS AND PROPORTIONAL GOMPERTZ HAZARDS MODELS

All 3 rough-and-ready translation tools assume that from some starting age/time onwards, the ratios of the age-specific mortality rates in the index and reference groups are approximately constant over this time span—or that the rate functions are separated by an approximately constant vertical distance when they are plotted on a log scale (as in the male:female and international contrasts shown in Web Figure 1, available online at https://doi.org/10.1093/aje/kwab178). Since statistical tests of this proportional hazards assumption miss/detect large/small deviations if the numbers of events are small/large, we are reluctant to be guided solely by such tests. We also look for biological analogies, such as the circumstances that lead to the "HR constancy over age/time" seen in male:female HRs: One might attribute this constancy to constitutional or life-history differences or other host differences that predate cohort entry and that can be expected to continue throughout the follow-up time. Although our main focus is on longevity and follow-up spanning 30–50 years, the short-term (daily) mortality rates in persons who become infected by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) also exhibit a close-to-constant male:female ratio in the first 60 days postdiagnosis (15).

Two of the 3 translation tools also assume proportional Gompertz hazards. In this model, from the starting age onwards, the logarithms of the age-specific all-cause mortality rates within the index and reference groups are—in addition to being parallel to each other (the proportional hazards model)—each approximately linear in age. This log-linear-in-age pattern in mortality rates was first noticed by Gompertz (6), but it was Thomas Edmonds who, with "great(er) ingenuity, neatness, and effect" (16, p. 8), promoted the same law in his book (17) and introduced the term "*force of mortality*" (18). Both regarded it as a continuous function of age and expressed it as number of deaths per person-year. As Edmonds put it, "the continuous change in the force of mortality is subject to three simple laws of geometric progression, corresponding to three remarkable periods of life" (17, p. v). The third of these—the one of interest here—Edmonds took to begin with the cessation of procreativity. For convenience, we will, somewhat arbitrarily, take it to start at the age of 45 years ($a_0$). And we will stay with the actuarial notation $m$ for mortality rate, rather than the more biostatistical $h$ or λ (introduced by Cox) for hazard rate. Gompertz and Edmonds found the force of mortality ($m$) during this third period of life, at all its ages ($a$), to be "expressible by a logarithmic curve" (17, p. v); that is, log($m[a]$) was linear in $a$. Thus, they expressed the mortality rate at age $a$, which we write as $m[a]$, as

$$m\,[a] = m\,[a_0] \times e^{b\times(a-a_0)},$$

where $m[a_0]$ is the mortality rate at age $a_0$ and $b$ is the slope when log($m$) is plotted against $a$.

From the empirical relationship between the log mortality rate and age (the intercept and slope are the estimates for $m[a_0]$ and $b$) in the regional mortality tables of the time, both Gompertz and Edmonds derived slopes of approximately 0.08. Thus, the fitted rate in the small interval centered on people's 46th birthday was $e^{0.08\times 1} = 1.083$ times (i.e., 8.3%) higher than that at their 45th birthday, and so on. As we see from more recent period mortality rates in 12 selected countries in the Human Mortality Database (19)—plotted in Web Figure 1—not only do the log rates continue to be linear but the ones for males and females are sufficiently parallel to justify the proportional hazards assumption, which is widely used in epidemiologic studies to produce a single (age-independent) HR. However, today's Gompertz' and Edmonds' "slopes" are closer to 0.10 or even 0.11. For the convenience of dealing with rounder numbers, just as Spiegelhalter does (4), we will adopt a modern-day "Gompertz slope" of 0.10.

The male:female all-cause mortality data in Web Figure 1 represent 1 example where—for our purposes here—the "same age slope" for the log rates in each of 2 compared cohorts provides an adequate measure. Spiegelhalter has described what we call the "proportional Gompertz hazards model" this way:

> *This means that the average chance of dying before a next birthday increases by around 10% for each year of aging, whether a man or a woman and regardless of age, if over 30. Equivalently, the average risk of dying before a next birthday doubles roughly every 7 years. Demographers have concluded that this ratio, 1.1, seems to be remarkably constant across populations and over time* (4, p. 3).

In support of his statement, Spiegelhalter cited a review paper by the eminent demographer Vaupel (20). In addition to the already mentioned constitutional, life-history, and sex differences, this "HR constancy over follow-up ages and sexes" may also reflect 1) the inclusion of *all* causes of mortality, so that 1 specific cause does not dominate; 2) the absence of identifiable vulnerable subgroups; and 3) the absence of any specific or immediate life-threatening conditions. Before employing the proportional-hazards–based or proportional-Gompertz-hazards–based tools, end users are urged to carefully consider whether these same types of conditions apply to the contrast in question.

We now consider each tool in turn.

## PROPORTIONAL-GOMPERTZ-HAZARDS–BASED AGE/RATE SHIFTING

The logarithm of 2 is approximately 0.7. Thus, if—as Spiegelhalter does—we use a Gompertz slope of 0.10, so that $e^{(7\times 0.10)}$ is approximately 2, then in the third stage of life, today's all-cause mortality rates at age $a+7$ years are approximately "double" those at age $a$ (a common shortcut for calculating the doubling time of invested money is to divide 70 by the compound interest rate). This geometric progression gives us a way to reexpress the HR or SMR of 1.3 (contrasting the mortality rates in the NFL (index) with the MLB (reference) category of players), as an "age shift" or a "rate shift."

The mortality rate for MLB players is

$$m\,[a|\text{MLB}] = m\,[45|\text{MLB}] \times e^{b\times(a-45)},$$

and the mortality rate for NFL players is

$$\begin{aligned} m[a|\text{NFL}] &= m\,[a|\text{MLB}] \times 1.3 \\ &= m[45|\text{MLB}] \times e^{b\times\left[(a-45)+\log(1.3)/b\right]}. \end{aligned}$$

Substituting log(1.3)/0.10 = 0.262 × 10 = 2.62, we obtain

$$m\,[a|\text{NFL}] = m\,[a + 2.62|\text{MLB}]\,.$$

The mortality rates of NFL players are comparable to the mortality rates of MLB players who are 2.62 years older: NFL players are 2.62 years "older," actuarially speaking, than MLB players. In Spiegelhalter's terminology,

$$\text{Age shift} = \log(\text{HR})/0.1 = 10 \times \log(\text{HR})$$

(2.62 years in this example) represents the difference in effective age.

However, this does not mean that if players in the 2 categories begin follow-up at a *common* age but are subject to differing (shifted) mortality rates (as in Figure 1A), the difference in RLE will also equal 2.62 years.

## FROM AGE SHIFT TO LONGEVITY DIFFERENCE

In fact, as other authors have pointed out (5, 8, 9), the difference in RLE must be less than the 2.62-year age shift. How much less depends on the starting age, $a_0$:

$$\begin{aligned} &\text{Difference in RLE} = 10 \times \log(\text{HR}) \\ &\qquad \times \text{shrinkage factor}, \end{aligned}$$

which depends on $a_0$—a little less if the starting age is 45 years (approximately 99% of those alive at age 45 will still be alive at age 47.62) but a lot less if it is 85 years (only about 75% of those alive at age 85 will be alive at age 87.62).

A concrete example illustrates the amount (the shrinkage factor) by which the effective age has to be multiplied to reduce it to the RLE. In the complete life table based on mortality rates in US males in 2017 (21), we can study the patterns in the RLEs at successive ages.

Table 1 shows that an $x$-year age shift in rates does not translate to an $x$-year difference in remaining life expectancy. For example, the RLE at a man's 45th birthday is 34.2 years, while the RLEs at his 46th, 47th, and 48th birthdays are 33.3, 32.4, and 31.5 years, respectively—that is, the RLE differences are 34.2 − 33.3 = 0.9 years, 34.2 − 32.4 = 1.8 years, and 34.2 − 31.5 = 2.7 years, or approximately 90% of the 1-, 2-, and 3-year age shifts. Thus, the 2.62-year wait ($\Delta_a$ = 2.62) until the mortality ratio shifts from 1.0 to 1.3 is

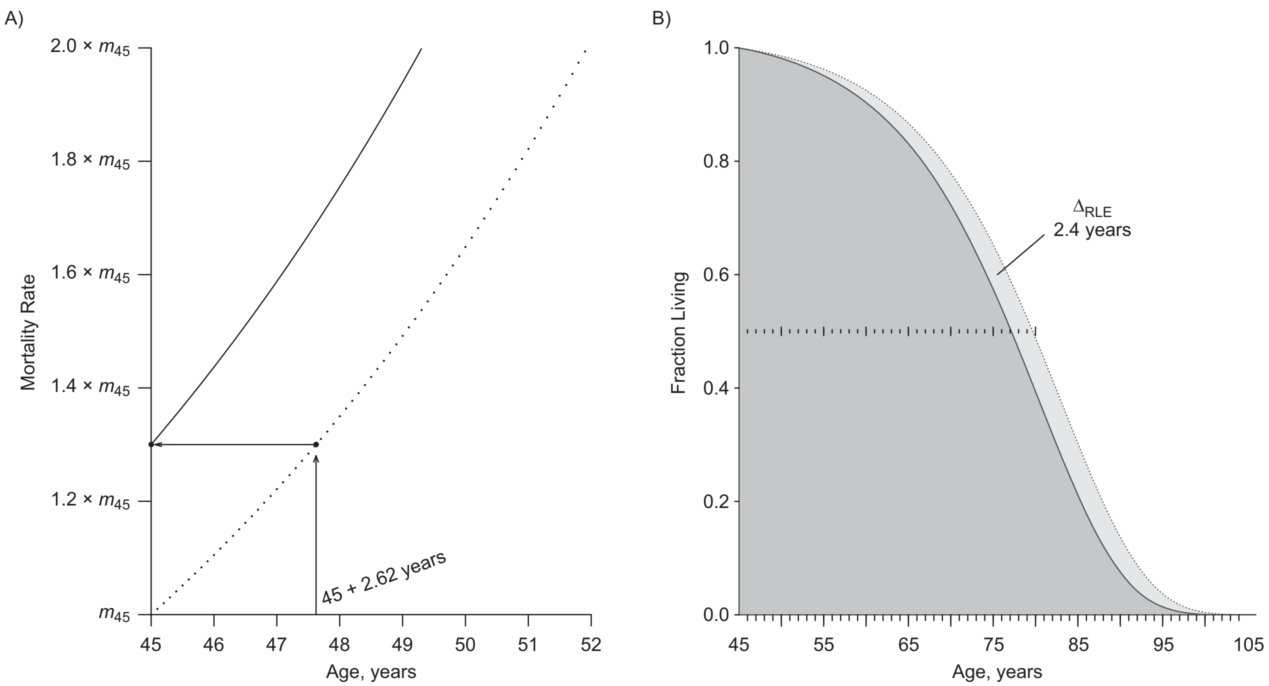


**Figure 1.** Relationship between age shift and the difference in remaining life expectancy in a hypothetical example where the hazard ratio is 1.3 and Gompertz' law applies. A) The mortality rate ($m$) in the reference category is shown in the lower curve (dotted line), starting at the value $m_{45}$ at age 45 years. If the Gompertz slope is 0.10, then for persons in the reference category the mortality rates at age $a + 7$ years are approximately double those at age $a$, and those at age $a + 2.62$ years are 1.3 times' those at age $a$. The upper curve (solid line) represents an index category where the age-specific mortality rates are 1.3 times' those in the reference category. B) A longevity comparison of the reference category (dotted line) versus the index category (solid line), beginning at a common age (age 45 years here). The remaining life expectancies (RLEs) of the contrasted categories (the areas under the respective survival curves) differ by 2.4 years. The median remaining duration of life (length of the horizontal hatched line) differs by a slightly different amount.

more appropriately converted to an RLE difference of 90% of 2.62—that is, to about 2.4 years (Figure 1B).

Therefore, even if Gompertz' law is a good fit, converting this 2.62-year shift in mortality rates to the difference in RLE requires further specificity. Some sense of the magnitude of the shrinkage can be found in Haybittle's article (7). He arrived at the log(HR)/(Gompertz slope) formula by starting from a previously established approximation (7) of the RLE. That approximation included a term that involved the force of mortality at the starting age divided by the Gompertz slope. By omitting this term from that formula, Haybittle found that when one starts from age 25 years, where the omitted term is small, this computed age shift provides a close approximation of the difference in RLE. Above that age, it increasingly overestimates the difference (5, 7, 8).

Web Figure 2 shows how the difference in RLE (a difference of mean values), expressed as the shrinkage factor (i.e., as a percentage of the age shift $\Delta_a$), is a function of the starting age, $a_0$. The pattern is similar to what Haybittle found. For example, if we measure from age 45 years, the difference in RLE is approximately 90% of the 2.62-year age shift—that is, approximately 2.4 years. Differences in the percentiles of the contrasted longevity distributions (22–24) are addressed in the Web Appendix.

## PROPORTIONAL-HAZARDS–BASED "PROBABILITY OF BEING OUTLIVED" METRIC

Some people may have difficulty imagining a remaining longevity contest between themselves and a sibling 2.62 years younger. They might prefer a contest between themselves and a lower–risk-profile twin and wish to know the probability/odds that they will die before that counterfactual twin. It has long been (but is not widely) known that this probability/odds is an even simpler function of the HR and that the conversion applies to any 2 distributions whose hazard functions are proportional (13). A proof is given in the Web Appendix.

$$\text{Probability that the higher-risk person dies first} = \text{HR}/(\text{HR}+1);\ \text{odds} = \text{HR}:1.$$

This simpler conversion does not force the translator to choose between the mean and some quantile. However, as Spruance remarked (when dealing with a desired clinical outcome), its greater generality (its nonreliance on equal-slope Gompertz distributions) comes at a cost: "When the hazard ratio is thought of as the odds that a patient will heal faster with treatment, a unitless term not directly reflective

**Table 1.** Life Table Entries for Males at Certain Ages, United States, 2017[a]

| Age Group, years | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25–28 | | | | 45–48 | | | | 65–68 | | | |
| $a$[b] | $a+1$ | $q$[b], %[c] | $RLE_a$[d] | $a$ | $a+1$ | $q$, % | $RLE_a$ | $a$ | $a+1$ | $q$, % | $RLE_a$ |
| 25 | 26 | 0.161 | 52.4 | 45 | 46 | 0.328 | 34.2 | 65 | 66 | 1.594 | 18.0 |
| 26 | 27 | 0.166 | 51.4 | 46 | 47 | 0.352 | 33.3 | 66 | 67 | 1.703 | 17.3 |
| 27 | 28 | 0.171 | 50.5 | 47 | 48 | 0.380 | 32.4 | 67 | 68 | 1.819 | 16.6 |
| 28 | 29 | 0.176 | 49.6 | 48 | 49 | 0.415 | 31.5 | 68 | 69 | 1.948 | 15.9 |

Abbreviation: RLE, remaining life expectancy.
[a] Data were obtained from the Centers for Disease Control and Prevention's 2017 life table for males (21).
[b] $a$, age; $q$, conditional probability of dying.
[c] Conditional probability of dying between ages $a$ and $a+1$.
[d] Expectation of remaining life at age $a$.

of the fundamental time units of the study, it also becomes more evident that the hazard ratio cannot convey information about how much faster this event may occur. The difference between hazard-based and time-based measures is analogous to the odds of winning a race and the margin of victory" (25, p. 2787).

## "GOMPERTZ TRANSLATE" IN PRACTICE

Our first example was to translate the SMR or HR of 1.3. If one is content with a very rough translation and wishes to emphasize age, one can use Spiegelhalter's age shift:

$$\Delta_a = \log(1.3)/0.10 = 10 \times \log(1.3) = 2.62 \text{ years.}$$

It has an easily remembered form when using a Gompertz slope of 0.1, and it provides an upper bound on the RLE difference. If one is uncomfortable providing an overestimate of the longevity difference, one can correct it downwards using factors derived from the broad RLE patterns observed in life tables and in Web Figure 3: For example, one might reduce $\Delta_a$ by, say, 10%, 20%, and 30% if the remaining life years begin at $a_0$ = 45, 55, and 65 years, respectively.

Provided that the proportional hazards assumption holds, we can forego the assumption of common-slope Gompertz distributions and simply calculate that the probability that an NFL player will be outlived by (will die before) an MLB player is $1.3/(1.3 + 1) = 0.57$ or 57%, or that the odds of this are 1.3:1.

Our second example was the headline that prompted us to take up this issue, namely the SMR of 0.76 contrasting the all-cause mortality rates of baseball players with those of average American men. If, again, we use the "Gompertz" slope of 0.10, then the 0.76 yields an age shift of

$$\Delta_a = 10 \times \log(0.76) = -2.7 \text{ years.}$$

This suggests that these baseball players were "actuarially younger," by 2.7 years, than average American men. The SMR of 0.76 is based on the mortality rates among 10,451 MLB players during the years 1979–2013 inclusive. From the database from which these players were identified, we estimate that their average age when their follow-up began was approximately 40 years. Thus, using the broad patterns in Table 1 and Web Figure 3, we might consider the MLB players to live an average of 2.4 years longer ($2.7 \times 0.9 = 2.4$) than average American men.

## THE SMALL PRINT—ENLARGED

We hesitate to emphasize exact correction factors, since the uncorrected $\Delta_a$ is already based on a "conveniently round number" slope of 0.10 (4). If we were to use a slope of 0.11, so that $1/0.11 \approx 9$, then the uncorrected $\Delta_a$ would be approximately $9 \times \log(0.76) = 2.5$ years.

We are also aware that any reported HR or SMR of 1.30 (such as in the NFL vs. MLB comparison) is just an average over the ages studied. It might be that if one could examine the raw data, one might find that it was 1.50 at age 45 years and 1.10 at age 75 years. One might also find that the slope in the experience in the reference category differed somewhat from the 0.10 or 0.11 considered here, or that the elevated HR is due to a specific cause of death or is time-limited. Since few publications report comparisons at this level of detail, all translations are at best approximate.

If the HR exceeds 1 by just a small amount—that is, if HR = 1.1 or 1.2, for example—then one does not need to reach for a calculator: $\log(\text{HR}) \approx \text{HR} - 1$. Likewise, if HR lies below 1 by just a small amount—if, say, HR = 0.9 or 0.8—then $\log(\text{HR}) \approx 1 - \text{HR}$. Thus, HR = 1.1 and HR = 0.9 correspond to (uncorrected) life-shortenings and life-extensions of approximately 1 year. One needs to calculate (rather than approximate) log(HR) if the HR is further from the null.

While Gompertz' law is a reasonable model for all-cause mortality in a general population, it cannot be expected to fit cause-specific mortality rates or subpopulations (patients) with newly diagnosed or known life-threatening conditions;

nor can the constant HR assumption be expected to apply throughout the entire life span. And even if it did, we would need 3 slopes—for the 3 different life phases that Edmonds described.

In those applications where 2 equal-slope Gompertz distributions are reasonable, the age shift allows us to accompany the probability of being outlived with a longevity difference.

## CONCLUSION

In closing, converting HRs to time ratios is considerably more complex than inverting currency exchange rates. Had the press office correctly inverted the 0.76 HR, the $1/0.76 = 1.32$ time ratio (implying that MLB players live 32% longer than US males ($100 \times (1.32 - 1) = 32\%$)) would have applied only if lifetimes followed exponential distributions. However, the only lifetimes that follow such distributions are those of inanimate objects (such as electronic components, or glassware) that are ended by external events (such as electrical surges, or being accidentally dropped) unrelated to the ages of the objects. This review promotes 2 conversion efforts better suited to human longevity studies.

An early and successful use of the concept of "effective age" was a dramatized 1973 television public service announcement "showing a 60-year-old Swede jogging effortlessly beside a puffing 30-year-old Canadian" (26). It is still considered a "major breakthrough in the conscience of Canadians" (26–28). However, uptake of the "metaphor" has been slow, as has the use of RLE differences developed by demographers. In linking these 2 concepts, our aim was to encourage the translation of HRs and SMRs derived from epidemiologic studies that focus on all-cause mortality. The "effective age" emphasizes years *already* "lost or gained off chronological age" (4, p. 1), while the RLE difference focuses directly on the *remaining* life years. Either way, we hope that our call to "Gompertz translate" improves the statistical reporting of mortality rate ratios or HRs and helps readers understand what they mean—even if the translation is approximate and postpublication. If one is unwilling to trust the equal-slopes assumption implicit in "Gompertz translation," one can fall back on the less informative "probability of being outlived" metric, which requires only the proportional hazards assumption.

In contrast to demographic studies of entire populations, epidemiologic studies of individuals have to deal with considerable selection, confounding, and measurement errors that preclude the much more refined calculations and statistical precision used in demography. These several sources of noise, along with the fact that neither Gompertz' law nor the proportional hazards assumption applies exactly in any application, mean that "translators" should not be overly precise. We suggest, as Spiegelhalter does in his Table 1 (4), that they round all age shifts or longevity differences to the nearest integer. It would also be appropriate to convert a (say) 57% probability of being outlived to (say) 55% or 60% so as to avoid false precision.

## ACKNOWLEDGMENTS

Author affiliations: Department of Epidemiology, Biostatistics and Occupational Health, Faculty of Medicine and Health Sciences, McGill University, Montreal, Quebec, Canada (Menglan Pang, James A. Hanley).

This work was funded by the Canadian Institutes of Health Research.

We thank numerous colleagues and students for their suggestions.

Conflict of interest: none declared.

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