The actual distributions of cells are given below, and compared with those calculated on the supposition that they are random samples from a population following the law which we have investigated: the probability P of a worse fit occurring by chance is then found.

I. Mean $=\cdot 6825$: $\mu_2=\cdot 8117$: $\mu_3=1\cdot 0876$.

Containing	0	1	2	3	4	5 cells
Actual	213	128	37	18	3	1
Calculated	202	138	47	11	1·84	·24
					2 (4 and 5 combined)	

Whence $\chi^2=9\cdot 92$ and $P=\cdot 04$.

Best fitting binomial $(1\cdot 1893-\cdot 1893)^{-3\cdot 6054}\times 400$ for which $P=\cdot 52$.

II. Mean $=1\cdot 3225$: $\mu_2=1\cdot 2835$ μ_3 : $=1\cdot 3574$.

	0	1	2	3	4	5	6
Actual	103	143	98	42	8	4	2
Calculated	106	141	93	41	14	4	1

Whence $\chi^2=3\cdot 98$ and $P=\cdot 68$.

Best fitting binomial $(\cdot 97051+\cdot 02949)^{46\cdot 2084}\times 400$ for which $P=\cdot 72$.

III. Mean $=1\cdot80$: $\mu_2=1\cdot96$: $\mu_3=2\cdot529$.

	0	1	2	3	4	5	6	7	8	9
Actual	75	103	121	54	30	13	2	1	0	1
Calculated	66	119	107	64	29	10	3		1	

Whence $\chi^2=9\cdot03$ and $P=\cdot25$.

Best fitting binomial $(1\cdot0889-\cdot0889)^{-20\cdot2473}\times400$ for which $P=\cdot37$.

IV. Mean $=4\cdot68$: $\mu_2=4\cdot46$: $\mu_3=4\cdot98$.

	0	1	2	3	4	5	6	7	8	9	10	11	12
Actual	0	20	43	53	86	70	54	37	18	10	5	2	2
Calculated	4	17	41	63	74	70	54	36	21	11	5	2	1

Whence $\chi^2=9\cdot72$ and $P=\cdot64$.

Best fitting binomial $(\cdot9525+\cdot0475)^{98\cdot53}\times400$ for which $P=\cdot68$.

These results are given graphically in Diagram II. on the next page.

It is possible to fit a point binomial from the mean and the 2nd moment according to the two equations $\mu_1'=nq$, $\mu_2=npq$ and these point binomials fit the observations better than the exponential series, but the constants have no physical meaning except that $nq=m$. And since the exponential series is a particular form of the point binomial and is fitted from one constant, while two are used for the "ad hoc" binomial, this better fit was only to be expected.

It will be noticed that in both I and III the 2nd moment is greater than the mean, due to an excess over the calculated among the high numbers in the tail of the distribution. As was pointed out before, the budding of the yeast cell increases these high numbers, and there is also probably a tendency to stick together in groups which was not altogether abolished even by vigorous shaking.

In any case, the probabilities ·04, ·68, ·25 and ·64, though not particularly high, are not at all unlikely in four trials, supposing our theoretical law to hold, and we are not likely to be very far wrong in assuming it to do so.

Let us now apply it to a practical problem: for some purposes it is customary to estimate the concentration of cells and then dilute so that each two drops of the liquid contain on an average one cell. Different flasks are then seeded with one drop of the liquid in each, and then "most of those flasks which show growths are pure cultures."

The exact distribution is given by

$$e^{-\frac{1}{2}}\left(1+\frac{1}{2}+\frac{(\frac{1}{2})^2}{2\,!}+\frac{(\frac{1}{2})^3}{3\,!}+\dots\right),$$

which is

No. of Yeast cells	*0*	*1*	*2*	*3*	*4*
Percentage Frequency	60·65	30·33	7·58	1·26	·16

or approximately three-quarters of those which show growth are pure cultures.

DIAGRAM II. Distribution of 400 Squares.

Firm lines: Actual Observations. Broken lines: Calculated from the Exponential Series. Where they coincide the firm line alone is given.

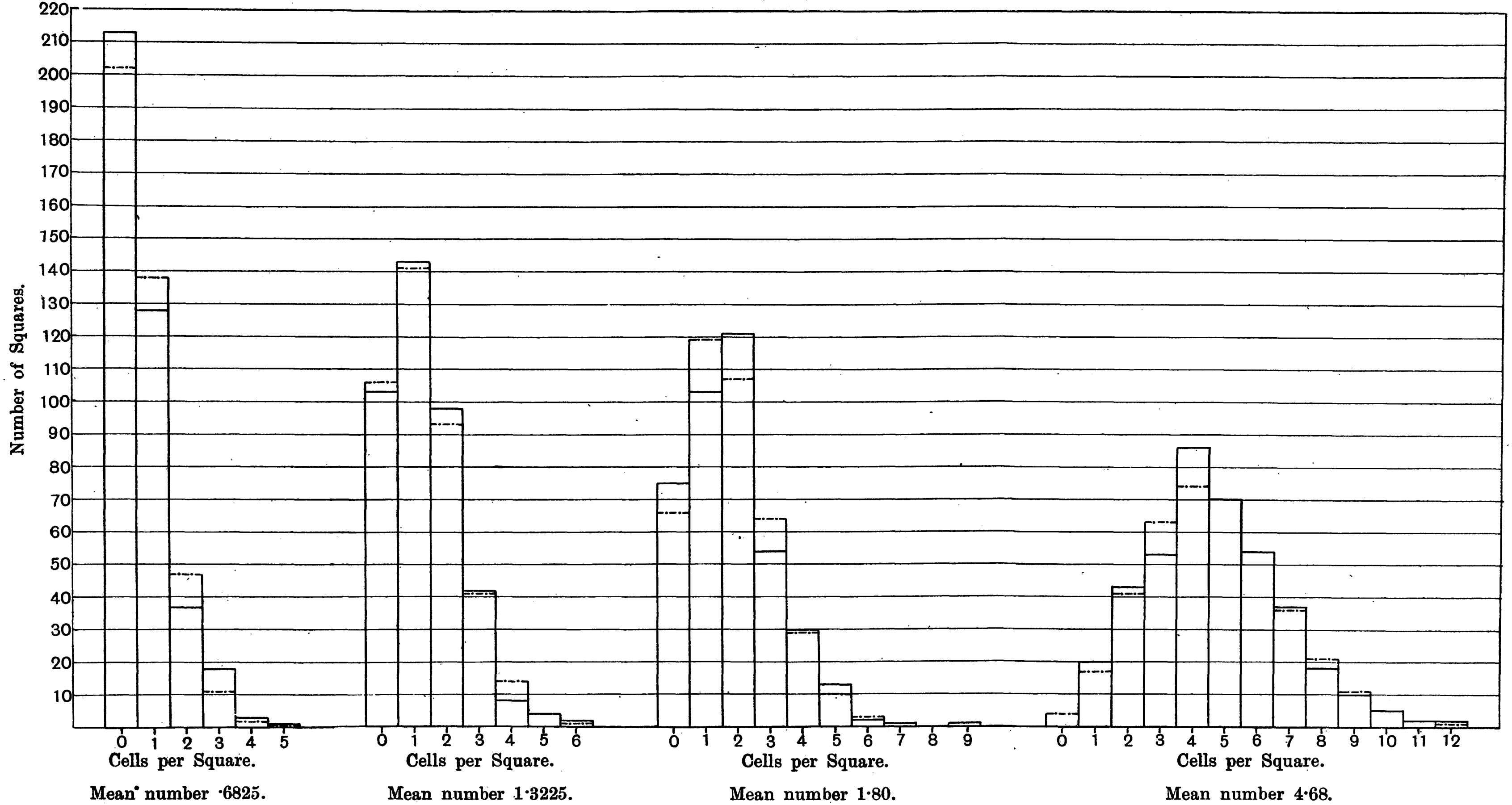

TABLE I.

Distribution of Yeast Cells over 1 sq. mm. divided into 400 squares.

2	2	4	4	4	5	2	4	7	7	4	7	5	2	8	6	7	4	3	4
3	3	2	4	2	5	4	2	8	6	3	6	6	10	8	3	5	6	4	4
7	9	5	2	7	4	4	2	4	4	4	3	5	6	5	4	1	4	2	6
4	1	4	7	3	2	3	5	8	2	9	5	3	9	5	5	2	4	3	4
4	1	5	9	3	4	4	6	6	5	4	6	5	5	4	3	5	9	6	4
4	4	5	10	4	4	3	8	3	2	1	4	1	5	6	4	2	3	3	3
3	7	4	5	1	8	5	7	9	5	8	9	5	6	6	4	3	7	4	4
7	5	6	3	6	7	4	5	8	6	3	3	4	3	7	4	4	4	5	3
8	10	6	3	3	6	5	2	5	3	11	3	7	4	7	3	5	5	3	4
1	3	7	2	5	5	5	3	3	4	6	5	6	1	6	4	4	4	6	4
4	2	5	4	8	6	3	4	6	5	2	6	6	1	2	2	2	5	2	2
5	9	3	5	6	4	6	5	7	1	3	6	5	4	2	8	9	5	4	3
2	2	11	4	6	6	4	6	2	5	3	5	7	2	6	5	5	1	2	7
5	12	5	8	2	4	2	1	6	4	5	1	2	9	1	3	4	7	3	6
5	6	5	4	4	5	2	7	6	2	7	3	5	4	4	5	4	7	5	4
8	4	6	6	5	3	3	5	7	4	5	5	5	6	10	2	3	8	3	5
6	6	4	2	6	6	7	5	4	5	8	6	7	6	4	2	6	1	1	4
7	2	5	7	4	6	4	5	1	5	10	8	7	5	4	6	4	4	7	5
4	3	1	6	2	5	3	3	3	7	4	3	7	8	4	7	3	1	4	4
7	6	7	2	4	5	1	3	12	4	2	2	8	7	6	7	6	3	5	4

TABLE II.

"Centre" Squares.

"Adjacent" Squares	1	2	3	4	5	6	7	8	9	10	11	12	Totals
1	6	6	9	15	15	9	4	3	2	—	—	—	69
2	6	14	17	31	24	17	10	5	6	2	1	1	134
3	8	15	25	32	37	20	15	7	7	1	4	—	171
4	18	34	33	45	48	41	22	7	5	4	1	—	258
5	15	24	37	47	39	37	18	12	11	4	1	2	247
6	9	17	25	39	34	32	14	8	2	4	1	1	186
7	5	12	14	21	19	16	9	7	3	—	—	—	106
8	3	5	7	8	12	8	6	1	3	4	—	—	57
9	2	6	7	5	10	2	2	3	—	1	—	—	38
10	—	1	1	4	4	4	—	3	—	1	—	—	18
11	—	1	4	1	1	1	—	—	—	—	—	—	8
12	—	1	1	—	1	1	—	—	—	—	—	—	4
Totals	72	136	180	248	244	188	100	56	40	20	8	4	1296

Mean of "Centre" Squares, 4·6821 ; S. D., 2·139.

Mean of "Adjacent" Squares, 4·7014 ; S. D., 2·116.

$r = +\cdot 016 \pm \cdot 037$.

Correlation table between the number of cells in a square and the numbers of cells in the four adjacent squares taken all over Table I.