CHARACTERIZATION OF DISTRIBUTIONS THROUGH CONDITIONAL EXPECTATION OF FUNCTION OF GENERALIZED ORDER STATISTICS

A. H. Khan

Department of Statistics and Operations Research Aligarh Muslim University Aligarh – 202 002, INDIA

Email: ahamidkhan@rediffmail.com

Let X_1, X_2, \cdots be a sequence of independent and identically distributed (*iid*) random variables (*rv*) with absolutely continuous distribution function (*df*) F(x) and probability density function

 $(pdf) \quad f(x), \ x \in (\alpha, \beta)$. Let $n \in N, \ n \ge 2, \ k > 0, \ \widetilde{m} = (m_1, m_2, \dots m_{n-1}) \in \Re^{n-1}, \ M_r = \sum_{j=r}^{n-1} m_j$, such

that $\gamma_r = k + (n-r) + M_r > 0$ for all $r \in \{1, \dots, n-1\}$. Then $X(r, n, \tilde{m}, k)$, $r = 1, 2, \dots, n$ are called gos if their joint *pdf* is given by

$$k \left(\prod_{j=1}^{n-1} \gamma_j\right) \left[\prod_{i=1}^{n-1} [1 - F(x_i)]^{m_i} f(x_i)\right] [1 - F(x_n)]^{k-1} f(x_n)$$

on the cone $F^{-1}(0) \le x_1 \dots \le x_n \le F^{-1}(1)$

Its variants among others are order statistics and record values. Using conditional expectations, which may not be necessarily linear, the form of distributions have been obtained when:

(i)
$$m_i = m_j$$

and (ii) $m_i \neq m_j, \gamma_i \neq \gamma_j$

Various deductions on characterization of distributions through *gos*, order statistics and records for varied distributions are discussed.