THE CALCULATION OF SINGULAR MULTIVARIATE NORMAL DISTRIBUTION FUNCTIONS

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We present a method to calculate multivariate normal distribution functions with singular covariance matrices. Miwa *et al.* (*JRSS*-B, Vol. 65, 2003) showed how multivariate normal distribution functions with *non-singular* covariance matrices can be accurately evaluated by expressing them as differences of a finite number of multivariate distribution functions with tridiagonal covariance matrices. It was essential in their method, however, that the covariance matrix should be non-singular.

In this paper we consider an n-dimensional multivariate normal random variable with any *singular* covariance matrix whose rank is m ($m < n$). It follows that the n-dimensional distribution function is equal to the probability volume of a polyhedron in the m-dimensional space. We shall show that any polyhedron can be expressed as differences of m-dimensional polyhedral cones each of which can be evaluated by the procedure proposed by Miwa *et al.* (2003). This allows us to evaluate accurately any singular multivariate normal distribution function.

Our procedure can be directly applied to many multiple comparison procedures, such as the modified Williams test or a Studentised range test of correlated treatment means, where the calculations of singular multivariate normal distribution functions are required.