STATISTICAL INFERENCE FOR A PREDATOR-PREY DYNAMICAL SYSTEM

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Differential equations (DIFE′s) are used to model the rate of change of a process defined of time, space, or some other continuum. Gregor F. Fussmann and his colleagues published a set of nonlinear DIFE′s that predicts correctly the qualitative dynamical behavior of a predator-prey food chain, that is, extinction, equilibria and cycles. The authors proposed one novel method to estimate parameters θ in these DIFE′s from their experimental data. DIFEs with the estimated parameters $\hat{\theta}$ can fit the experimental data well in quantity, even when two out of four components are not observed at all. The idea is to smooth data by a combination of a set of basis functions, penalized by its fidelity to DIFE′s. A smoothing parameter λ reconciles the trade-off between fitting the data and fidelity to DIFE′s. This process is called *the L-splines smoothing*. For any given parameters θ, a coefficients vector $\mathbf{c}$ to basis functions can be estimated by the L-splines smoothing. The dimension of parameter space is reduced by treating the coefficient vector $\mathbf{c}$ as the implicit function of parameters θ. Then the parameters θ can be estimated by maximizing the likelihood function, with the analytic expression for the gradient given by the Implicit Function Theorem. A byproduct of this method is that we can estimate the initial values and other missing data by the L-splines smoothing. The desirable smoothing parameter $\hat{\lambda}$ is obtained by minimizing the Stein′s unbiased risk estimate for the total prediction error.