XXXVI. *The Probability Distribution of the Time Intervals of α Particles with Application to the Number of α Particles emitted by Uranium.*

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IN counting the α particles from radio-active substances by the scintillation method the great preponderance of short intervals is very noticeable, especially when the scintillations are appearing at a slow rate. In fact this preponderance leads one at first sight to consider the α particles as coming in groups and not distributed according to a simple probability law. Rutherford and Geiger* have recently examined the emission of α particles from polonium from this point of view. In their experiments scintillations produced on a definite area of a zinc-sulphide screen placed in an exhausted vessel at a distance from the polonium source of α particles, were recorded on a travelling chronograph tape and analysed according to the numbers occurring in successive equal time intervals, generally large compared with the average time interval of the scintillations. The results were found to be in good agreement with Bateman's† theoretical formula that $x^n e^{-x}/n!$ is the probability of n scintillations occurring in a given interval where x is the true average number for the interval. However, were the particles given off in equal groups and the groups distributed in time according to probability it is conceivable that the above formula would still hold. It would, therefore, seem preferable in many respects to test the application of the probability laws to actual time intervals between successive α particles and to count the whole number from a given source instead of only those emitted within a relatively small solid angle.

The problem of the calculation of the distribution of time intervals can be approached in the following way. Let the average time interval between successive α particles falling on a zinc-sulphide screen from a radio-active source be $1/\mu$, and let us assume that the time under consideration is small compared with the time period of the radio-active substance. Assuming that at time 0 the observer sees a scintillation, let us find the probability that a time interval, t, elapses without

* Rutherford and Geiger, "Phil. Mag." XX., p. 698, Oct., 1910.

† *Loc. cit.* Note by H. Bateman.

the occurrence of a second scintillation but that within a further very small interval, δt, a scintillation occurs. This problem corresponds exactly to the method employed in practice. The probability that no scintillation occurs in the time interval t is $e^{-\mu t}$.* The probability that a scintillation occurs in the time interval from t to $t+\delta t$ is independent of t and is $\mu\delta t$. As these events are independent of one another the probability of an interval from t to $t+\delta t$ is the product of their separate probabilities and is therefore $\mu e^{-\mu t}\delta t$; or in a large number of N intervals the probable number of intervals larger than t and smaller than $t+\delta t$ is $N\mu e^{-\mu t}\delta t$.

This result is at first sight somewhat surprising,† for it indicates that whatever the value of μ small intervals are more probable than large ones, whereas one would at first sight expect that the intervals would be distributed according to a law somewhat similar to that of Maxwell for the distribution of the velocities of the molecules of a gas.

The above formula has been applied to several sets of observations made by us on the α particles from polonium and uranium and found to agree well with experiment. The case of uranium is particularly interesting, for Boltwood ‡ and Geiger and Rutherford § have shown that uranium emits two α particles. If these α particles are given off together or by successive α ray products, the second having a period less than a few seconds, then in observing scintillations caused by α particles from uranium placed in contact with a zinc-sulphide screen one would expect a larger number of short intervals than according to the simple theory above. The experimental arrangement used to test this point is shown in Fig. 1 and is the same as that of Geiger and Marsden‖ in their experiments on the α particles from actinium and thorium emanations. S_1 and S_2 are two

* H. Bateman, *loc. cit.* It is not at first sight clear that we are justified in applying this formula to a time interval commencing with a scintillation, but the time of occurrence of the scintillation is really arbitrary as referred to the times of previous scintillations. The result may also be obtained as follows: The probability that a time interval that does not contain a scintillation is $e^{-\mu t}$, and this is, therefore, the probability that an interval without scintillation is greater than t, or in N such intervals the number greater than t is $Ne^{-\mu t}$. Similarly, the number of intervals which are greater than $t+\delta t$ is $Ne^{-\mu(t+\delta t)}$. The difference gives us the number of intervals between successive scintillations greater than t but smaller than $t+\delta t$, and is $N\mu e^{-\mu t}\delta t$.

† *Cf.* Rutherford and Geiger, Roy. Soc. "Proc.," A. LXXI., p. 141 (1908).

‡ Boltwood, "Amer. Journ. Sci.," Vol. XXV., p. 270 (1908).

§ Geiger and Rutherford, "Phil. Mag.," XX., p. 692, 1910.

‖ Geiger and Marsden, "Phys. Zeit.," XII., p. 7, 1910.

screens of zinc-sulphide coated on glass with the zinc-sulphide coatings facing each other and separated by sheets of aluminium foil. One of these sheets was coated with a very fine layer of uranium-oxide produced by pouring on it uranium-oxide suspended in alcohol and allowing the alcohol to evaporate. Two microscopes, M_1 and M_2, were focussed on exactly opposite areas of the two screens and the scintillations observed were recorded by two separate observers on the same travelling tape of a Morse inker by means of two needle points arranged to puncture the tape by arrangements of levers. Ink marks were also made on the tape at second intervals by means of a circuit containing an electromagnet and a pendulum contact. Just sufficient sheets of thin aluminium foil were used that no scintillations on one screen were visible in the microscope focussed on the other, more than one being necessary on account of the difficulty of keeping it unpunctured by the crystals of the screen on subsequently pressing them together. When the

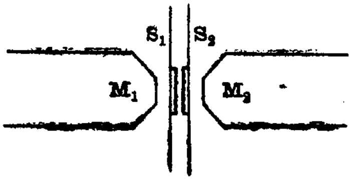

FIG. 1.

screens were placed together in this way they were found to be about $\frac{1}{3}$ mm. apart, while the diameters of the fields of the two similar microscopes were about 2·3 mm., so that practically the whole of the particles given off by the uranium between the screens would impinge on either one or the other of them. In making the observations the tape was started and scintillations recorded by both observers for a period of from two to five minutes. The tape was then stopped and after the eyes had been rested another set of observations was made, and so on, until a sufficient number of scintillations had been observed. The records on the tape were then examined and a curve drawn giving the number of occurrences of different intervals and this curve compared with that obtained from the formula given above. Thus in the particular case shown in Fig. 2.

Total time$=$2,988·5 seconds.

Number of intervals$=$(taking the observations on both screens) 319.

Average interval$=$9·37 seconds. $\mu=0{\cdot}1068$ (approx.).

The probability of an interval from 0 to 1 second is, to the approximation sufficient for the present experiment, the value of $N\mu e^{-\mu t}$ for $t=0{\cdot}5=319\times 0{\cdot}1068\ e^{-0{\cdot}0} \quad =32{\cdot}2$.

The actual number of such observations was 36.

In this way the probability curve shown in Fig. 2 has been drawn and it will be seen that the actual values observed, which are indicated by the points, agree with it to an extent within the limits of the probability error. The tape was also examined by the method of Rutherford and Geiger, the number of scintillations in successive intervals of 16 seconds being measured. The number of occurrences of each number are given in column I. of the following table, while in column II. are the numbers calculated from Bateman's formula.

No. of particles in interval of 16 seconds.	No. of groups.	
	I. Experimental.	II. Calculated.
0	35	34
1	56	58
2	49	49·5
3	32	28
4	9	12
5	5	4·1
6	1	1·15

The agreement is good, and considering the small number of observations it is probably fortuitous. Good agreement between experiment and theory was also found in other cases in which the amount of uranium was varied so as to give different rates of emission of α particles, while experiments in which a layer of polonium was used also showed excellent agreement with theory. The polonium was obtained by evaporating on an aluminium foil a solution of the active deposit on the walls of an old tube which had contained radium emanation. The emission of α particles was also examined by the foregoing methods on two separate screens placed at various distances up to 83 cm. from sources of polonium. Agreement between experiment and theory was in all cases satisfactory.

Through the kindness of Prof. Rutherford and Dr. Geiger, we have been allowed to examine some of the records of the scintillations made in their experiments on the "Probability Variations in the Distribution of α Particles." These also showed excellent agreement with the above theory.

The theory of these observations is independent of whether the screens are continuous or whether scintillations are only produced by a fraction of the α particles. However (with the screens used) it is unlikely that more than 10 per cent. of the α particles failed to produce scintillations. The chief source of error might be the missing of scintillations through temporary fatigue of the eyes, for in the case of uranium the range of the particles is only 2·7 cm. and their velocity consequently small, this being further cut down by the aluminium foils. Using the low magnification necessary to obtain a large field

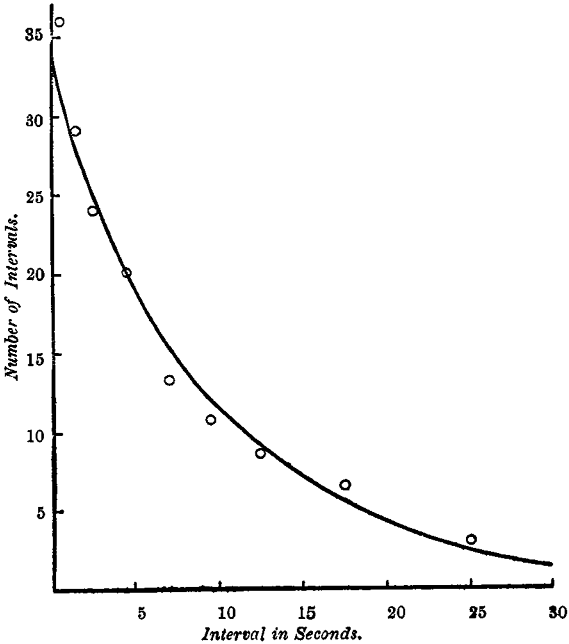

FIG. 2.

and, with the particular objectives used, the consequent small solid angle of the light from the scintillations, they were much fainter than in usual scintillation experiments. The missing of scintillations, however, is partly avoided by the observations of one observer forming a check on those of the other. To test this point further and to verify the experimental arrangement the observations of Geiger and Marsden* on the double scintillations from actinium emanation were repeated. The emana-

* *Loc. cit.*

tion was allowed to diffuse up between the two screens and the same numbers of connected scintillations were observed as in their experiments. Thus, with the emanation and active deposit in equilibrium nearly 50 per cent. of the scintillations appeared simultaneously on either the same or both screens, and a simple calculation showed that with the arrangement used only a few of the α particles could have been missed. As a further support of the theory it was found that when the doubles with actinium emanation were neglected and counted as single scintillations the distribution of the remaining time intervals was in agreement with theory.

We thus see that the variation of the intervals between the emission of successive α particles from both polonium and uranium shows good agreement with simple probability theory, and we therefore conclude that the two α particles from uranium are not given off simultaneously. Further, if they are given off by successive α-ray products the period of the second must be greater than a few seconds. This is also unlikely because of the small ranges of both sets of α particles.*

Further experiments are in progress to apply the foregoing method to the α particles from thorium B and thorium C and also to investigate the possible existence of a short-life product in the active deposit of actinium indicated by the 10 per cent. of connected particles found in the experiments of Geiger and Marsden.

We are indebted to Prof. Lees, F.R.S., for his kind interest in these experiments.

ADDENDUM.

[JULY 18, 1911.]

Prof. Rutherford and Dr. Geiger have very kindly allowed us to publish the figures relating to observations on some of the records of the scintillations obtained in their experiments. The time intervals between successive scintillations were deduced from the measured distance between the records on the tapes, allowance being made for the varying velocity of the tape by reference to the time marks. The time marks were made at 30 seconds intervals and the average distance between them was about 7 cm.

* A. Foch, "Le Radium," VIII., p. 101, 1911.

The results are given in the following table. Column II. gives the experimentally determined number of intervals of duration between the limits given in Column I. The theoretical numbers in Column III. were calculated as follows :—

Total number of intervals .... = 7,564
Total time .... =14,598 secs.
Average interval .... = 1·930 secs.

$$\mu = 0{\cdot}5181.$$

Number of intervals of duration greater than t_1 and less than

$$t_2 = \int_{t_1}^{t_2} \mathrm{N}\mu e^{-\mu t} dt = \mathrm{N}(e^{-\mu t_1} - e^{-\mu t_2}) = 7{,}564(e^{-0{\cdot}5181 t_1} - e^{0{\cdot}5181 t_2}).$$

Duration of interval between successive scintillations. Seconds.	Number of intervals.	
	Experimental.	Calculated.
0·0 to 1·0	3,106	3,059
1·0 to 2·0	1,763	1,822
2·0 to 3·0	1,115	1,085
3·0 to 4·0	658	646
4·0 to 5·0	389	385
5·0 to 6·0	206	229
6·0 to 7·0	130	136
7·0 to 8·0	86	81
8·0 to 9·0	42	48
9·0 to infinity	68	71

The agreement between the experimental and theoretical values is surprisingly good and is probably well within the probability error.