XVI. *Note on the Paper by Prof. E. Rutherford, F.R.S., and H. Geiger, Ph.D., on "The Probability Variations in Distribution of α Particles."* *By* E. C. Snow, *M.A.**

IN the Phil. Mag. for October last Prof. Rutherford and Dr. Geiger contributed a paper with the above title, a mathematical note by Mr. Bateman being appended. The latter showed that if x is the true average number of α particles falling on a screen from a constant source in a given interval, the probability that n α particles are observed in the same interval is given by $\frac{x^n}{n!} \cdot e^{-x}$ (p. 705). The figures found by the experimenters were compared with those obtained from the above expression, and the conclusion reached was that "on the whole, theory and experiment are in excellent accord," but no numerical measure of the agreement was attempted.

As the distribution of the figures is of a general form often reached in many branches of statistics, it is of interest to compare the numbers given in the paper with those derived from fitting an ideal frequency curve to the experimental results, and also to obtain a measure of the "goodness of fit" of the latter with the theoretical values.

The theory of ideal frequency curves has been developed by Prof. Karl Pearson †, and the notation used here is that devised by him.

Taking the figures given by the experiments for $\frac{1}{8}$ minute intervals (p. 701, last row but one) the following statistical constants are found :—

$$\mu_1 = 0.$$

$$\mu_2 = 3{\cdot}6114. \qquad \beta_1 = {\cdot}2451.$$

$$\mu_3 = 3{\cdot}3979. \qquad \beta_2 = 3{\cdot}5265.$$

$$\mu_4 = 45{\cdot}9922.$$

A reference to Rhind's Tables ‡ shows that the distribution lies just within the area of Pearson's Type IV., but is quite close to Type V. The equation of the first of these is of the form

$$y = y_0(x^2 + a^2)^{-m} e^{-r \tan^{-1} x/a},$$

and of the second

$$y = y_0 x^{-p} . \, e^{\gamma x}.$$

* Communicated by the Author.

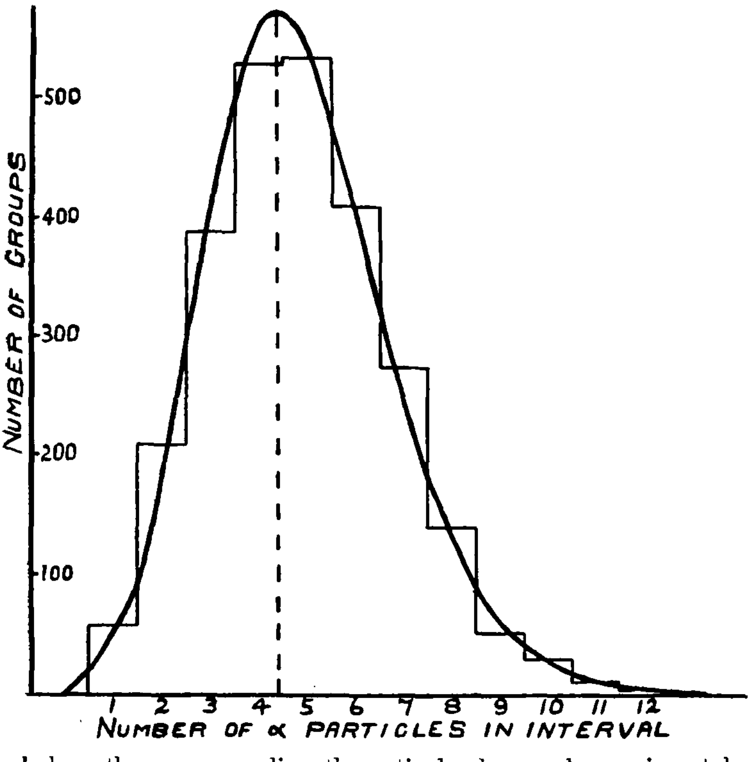
† Phil. Trans. vol. clxxxvi. A. pp. 343–414.

‡ 'Biometrika,' vol. vii. July and October 1909, p. 131.

As the latter is far more easy to draw than the former a curve of Type V. has been fitted to the data, the following being the extra constants involved :—

$$p = 70{\cdot}2582. \qquad \log y_0 = 116{\cdot}1842.$$
$$\gamma = +1063{\cdot}8192. \qquad \text{Origin} = \text{mean} - 15{\cdot}5852.$$
$$\text{Mode} = \text{mean} - {\cdot}4437,$$

the skewness being ·2335. The curve is shown in the diagram, the stepped figure giving the corresponding experimental frequencies. The areas of portions of the curve bounded by ordinates determined by the data are shown



below, the corresponding theoretical values and experimental numbers being also given. The last five observations are clubbed together as they could not accurately be measured separately by the planimeter.

Experimental values .	57	203	383	525	532	408	273	139	45	27	16
Theoretical values ...	54	210	407	525	508	394	254	140	68	29	21
Values from Curve...	57	216	399	553	533	391	251	123	49	24	12

For the "goodness of fit" * of the experimental values with the curve we find $\chi^2 = 9{\cdot}6270$ and $P = {\cdot}474$. For the theoretical values and the experimental values the figures are $\chi^2 = 13{\cdot}9826$ and $P = {\cdot}173$. In the latter case, therefore, in one trial out of six a random sampling would lead to a system of numbers diverging more widely from the theory than do the actually observed figures. The fit may be considered fair. In the former case the corresponding probability is one in two, or three times as good. But this is obtained by the use of extra constants and must not be taken as controverting the authors' conclusions that "as far as the experiments have gone, there is no evidence that the variation in number of α particles from interval to interval is greater than would be expected in a random distribution."

In the case of the $\frac{1}{4}$ minute intervals the "goodness of fit" of the two rows of figures (the one obtained from experiment and the other from theory) given at the bottom of p. 703 of the paper can also be ascertained. Here $\chi^2 = 14{\cdot}9953$ and $P = {\cdot}31$, the observations at the end of the range being clubbed together, as before.

Judging from the figures given, therefore, the agreement between theory and experiment in the case of the $\frac{1}{4}$ minute intervals is better than in the $\frac{1}{8}$ minute intervals, a conclusion opposite to that of the authors of the paper. This, however, can be accounted for by the very poor agreement between experiment and theory in the case of 8 particles per interval in the latter experiments. Here theory gave 68 while experiment only showed 45, and this discrepancy contributed no less than 7·78 to the total value (13·98) of χ^2.

The ideal frequency distribution could also be obtained from the figures given for the $\frac{1}{4}$ minute interval experiments. For these, $\beta_1 = {\cdot}2283$ and $\beta_2 = 3{\cdot}6429$, and the distribution has passed over into a true Type IV. curve, instead of being on the border-line between Type IV. and Type V. as in the case of the $\frac{1}{8}$ minute interval experiment.

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* See Pearson, Phil. Mag. vol. l. pp. 157–175, July 1900. Also "Tables for Testing the Goodness of Fit of Theory to Observation," W. Palin Elderton, 'Biometrika,' vol. i. pp. 155–163.