Miettinen OS. Theoretical Epidemiology
Principles of Occurrence Research in Medicine
Wiley 1985.

APPENDIX 4

Crude, Adjusted, and Standardized Rates

Rates characterize populations (cf. Appendix 1), and populations generally are quite inhomogeneous as to the commonality of any given phenomenon. Thus there is a need for rate concepts according to how the *overall* (unconditional) rate is constituted in the context of variation among *specific* (conditional) rates that characterize subdomains of the population.

A.4.1. CRUDE RATE AS A WEIGHTED AVERAGE OF SPECIFIC RATES

With reference to any particular population experience, or study base to be specific, the *actual* (empirical) overall rate r is referred to as the crude rate. Thus, a crude rate (CR) is simply

$$CR = \frac{c}{B}, \tag{A.4.1.}$$

where c is the total empirical number of cases and B is the size of the base (number of subjects S or amount of population time T).

EXAMPLE A.4.1. Consider the mortality data for male agricultural workers in Table A.4.1. In the age range considered therein, 14,000 deaths occurred

TABLE A.4.1.
Mortality data, actual and hypothetical, for males in England, 1951[a]

	Agricultural Workers			Hypothetical Occupational Group			
Age (yr)	Deaths in 1949–1953	Population in 1951	Rate in $(10^3y)^{-1}$	Deaths in 1949–1953	Population in 1951	Rate in $(10^3y)^{-1}$	National Rate in $(10^3y)^{-1}$
20–24	540	83,400	1.3	65	10,000	1.3	1.4
25–34	960	133,300	1.4	144	20,000	1.4	1.6
35–44	1,500	131,600	2.3	57	5,000	2.3	2.9
45–54	3,420	117,200	5.8	58	2,000	5.8	8.2
55–64	7,530	90,600	16.6	83	1,000	16.6	23.0
20–64	14,000	556,100			407	38,000	

[a] Cf. Schilling, 1973

in a dynamic population of (average) size 556,100 followed for the 5-year period of 1949–1953. Thus, the crude death rate (incidence density of death) was 14,000 cases in a 556,100(5*y*) space of population time (candidate time *T*), that is, $CR = 14{,}000/[556{,}100(5y)] = 5.0/(10^3y)$.

A *CR* has the structure of being a *weighted average* of the constituent specific rates, with weights equal (or proprotional) to the sizes of the respective subdomains of the actual base:

$$CR = \frac{\sum_j W_j(c_j/B_j)}{\sum_j W_j}, \qquad \text{(A.4.2)}$$

where

$$W_j = \frac{B_j}{B}.$$

This relation, which is fundamental to all understanding of rates, is a mere algebraic truism.

Example A.4.2. Recall Example A.4.1, where $CR = 5.0/(10^3y)$. From the specific rates in Table A.4.1 this crude value may be summarized as follows: $\{[83.4(1.3) + 133.3(1.4) + 131.6(2.3) + 117.2(5.8) + 90.6(16.6)]/(83.4 + 133.3 + 131.6 + 117.2 + 90.6)\}/(10^3y) = 5.0/(10^3y)$.

A *CR* thus reflects not only the specific rates of the various subdomains but also the relative sizes of the latter, through latent weights of a totally ad hoc nature. This makes such rates ill-suited for many purposes, particularistic as well as scientific.

A.4.2. ADJUSTMENT AND STANDARDIZATION: THE IDEAS

There is a need to separate the two elements in a *CR*—the set of specific *rates* on one hand and the set of their corresponding *weights* on the other.

The most elementary, yet thorough, way of coping with this need is to consider the set of specific rates, as in Table A.4.1. The drawback with this is complexity, difficulty with assimilation in the context of excess detail. Hence there *is* a need for overall rates, but with deliberately chosen weights, the "native" weights of the *CR* being but one among the options.

In an *adjusted* rate, the native weights (proportional to the base experiences themselves) are replaced by some other, external set of weights. This transforms the *CR*, the actual overall rate, into its equivalent in the context of a *hypothetical* structure of the base. The adjusted rate expresses what the overall *CR* would have been, had the base had the alternative structure and had the specific rates remained unchanged.

Example A.4.3. Recall the two examples presented above, with the crude overall death rate of $5.0/(10^3y)$ for male agricultural workers. One might ask—perhaps ill-advisedly (Wang and Miettinen, 1982)—how this rate compares with the male mortality in the nation at large. One would not wish this comparison to be clouded by the difference in age structure between the two populations; rather, the comparison ought to address, in an overall sense, the relative magnitudes of age-specific rates between the compared populations. To this end, one option is to adjust the national *CR* to the age structure of the agricultural workers. This means replacing the weights inherent in the national *CR* by ones proportional to the age-specific base sizes in the agricultural experience, that is, to the numbers of person years of observation by age in that hypothetical structure for the national experience. This adjusted national rate is $\{[83.4(1.4) + 133.3(1.6) + 131.6(2.9) + 117.2(8.2) + 90.6(23.0)]/556.1\}/(10^3y) = 6.8/(10^3y)$. The difference between this adjusted rate for the nation and the *CR* for the agricultural occupation is no longer attributable to the difference in age structure between the two.

When two or more rates involve a common set of weights, whatever this set may be, they are said to be *standardized*—meaning *mutually* standardized. This does not mean that the rates involved are all adjusted; a *CR* may be a member of a mutually standardized set of rates, as in Example A.4.3. The point is merely that the weights are the same, so that any difference(s) between (among) the rates is (are) not attributable to difference(s) in structure (weights), but must be a reflection of differences in the specific rates. (Lack of difference, however, does not mean that the values of the specific rates are the same for each of the compared populations.)

A.4.3. THE NOTION OF "INDIRECT" STANDARDIZATION

There are those who believe that there are two types of mutually standardized rate pairs or rate sets, "*directly*" and "*indirectly*" standardized. This is a misapprehension. As noted, the issue is singular, modification of weights, and the role of the "standard" is to supply those weights.

The notion of a duality of standardizations arises from the consideration of *observed* and *expected* numbers of cases in some base of interest. The observed number ($c_1 = O_1$) is the actual number, the numerator of the *CR* for this *index* experience. The "expected" number ($\hat{E}_1$) is hypothetical, the number that would have materialized in the index base had the specific rates of a *reference* population, such as those of the nation at large, obtained in the index experience. The ratio $O_1/\hat{E}_1$ characterizes the relative magnitudes of the rates in the index and reference experiences, indicating the relative size of the index rate in comparison with the reference rate, upon standardization for age or whatever. Thus, one may use the comparison

$$\frac{O_1}{\hat{E}_1}(CR_0) \text{ vs. } CR_0 \qquad \text{(A.4.3.)}$$

with the assurance inherent in standardization, namely that any difference is indicative of nonidentity of the set of specific rates between the index and reference experiences. It is $(O_1/\hat{E}_1)(CR_0)$ that is thought of as the "indirectly standardized" rate, with the "indirectness" meaning that the specific rates of the index experience never need to be considered. The observed number is the numerator of the *CR*, and the "expected" number involves only the structure of the index experience with the empirical rate elements derived from the reference experience:

$$\hat{E}_1 = \textstyle\sum_j B_{1j} r_{0j}. \qquad \text{(A.4.4.)}$$

Not only is this calculation thought to represent a special form of standardization, but it is also thought to be preferable to "direct" standardization when the index experience is small relative to the reference experience. After all, it focuses directly on the total number cases in the scarce index experience.

To gain insight into these notions, consider the structure of the comparison shown in Equation A.4.3 in terms of the elements that are relevant to standardization—the specific rates in the index experience, $\{r_{1j}\}$, those in the reference experience, $\{r_{0j}\}$, and the common set of weights, $\{W_j\}$. The essence of the formulation is the contrast between the observed and "expected" numbers, and it can be recast as

$$\frac{O_1}{B_1} \text{ vs. } \frac{\hat{E}_1}{B_1}. \qquad \text{(A.4.5.)}$$

The left-hand element is, evidently, the *CR* for the index experience and, recalling the structure of E_1 (Equation A.4.4), the right-hand element evidently is the reference rate standardized to the structure of the index experience. There is nothing "direct" or "indirect" about this standardization; it is just standardization, of the one and only kind. A point of note is, however, that the common *weights derive from the index experience.*

A.4.4. THE STANDARDIZED MORTALITY (MORBIDITY) RATIO

In the context of dynamic-population mortality (incidence density of death), the ratio of two rates standardized by the use of weights from the index experience is commonly termed *the standardized mortality ratio, the SMR.*

Sometimes the acronym is applied to morbidity density as well, with the same implication of uniqueness. Either way,

$$\begin{aligned} SMR &= \frac{O_1}{\hat{E}_1} \\ &= \frac{O_1/B_1}{\hat{E}_1/B_1} \qquad \text{(A.4.6.)} \\ &= \frac{\sum_j B_{1j} r_{1j}}{\sum_j B_{1j} r_{0j}}. \end{aligned}$$

Example A.4.4. Recall Example A.4.3. For the agricultural workers (the domain of express interest, the index domain), the observed number of deaths was 14,000. The corresponding expected number, had the age-specific national rates applied to the agricultural subpopulation as well, would have been 83,000(5y) [1.4/(10³y)] + 133,300(5y) [1.6/(10³y)] + . . . = 18,800. Thus, the observed-to-expected ratio for the agricultural workers, with the national population as the referent, was 14,000/18,800 = 0.74. This is also the ratio of the respective mortality rates, mutually standardized, with the index experience (that of agricultural workers) providing the common weights. Hence, the ratio involves the *CR* for agricultural workers, 5.0/(10³y) (cf. Example A.4.1), and the reference rate adjusted to the structure of the agricultural population, 6.8/(10³y) (cf. Example A.4.3). The ratio is 5.0/6.8 = 0.74, the $O/\hat{E}$ ratio.

Since the weights in an $O/\hat{E}$ ratio derive from the index experience, they are specific to each index category in the context of two or more ratios. Thus, even though each $O/\hat{E}$ ratio is *internally* standardized, a set of such ratios is *not mutually* standardized (Miettinen, 1972b). In other words, a difference between two $O/\hat{E}$ ratios does not indicate that there must be a difference between the respective sets of specific rates.

Example A.4.5. Consider again the data in Table A.4.1. The agricultural and the hypothetical occupational categories are characterized by identical age-specific rates. Thus, any comparable, mutually standardized, overall measures for those two experiences are identical. The $O/\hat{E}$ ratio for the agricultural experience is 0.74 (cf. Example A.4.4). For the hypothetical population it is 407/{10,000(5y) [1.4/(10³y)] + 20,000(5y) [1.6/(10³y)] + . . .} = 0.81. This value differs from the 0.74 for the agricultural experience, reflecting the incomparability of a set of $O/\hat{E}$ ratios rather than differences between the respective sets of specific rates.

Comparability among the values in a set of rate ratios presupposes the employment of a common set of internal standards for each. One possibility is to use the referent as the common standard as well. This means using the specific rates of the ith index category to compute the respective "expected" number in the reference category, $\hat{E}_{0i}$. The rate ratios, internally and mutually standardized, for the various compared categories are then $\{\hat{E}_{0i}/O_0\}$ (Miettinen, 1973).

A.4.5. PRECISION-MAXIMIZING WEIGHTS

When one attempts to maximize the *precision* of a single contrast of rates, a reasonable choice of the weights of the internal standard is to draw them from one of the compared experiences, the one in which the experience is more *sparse* (Miettinen, 1972a)—often the index experience. It is even better to employ a standard in which the weights are proportional to the respective amounts of comparative information among the subcategories (cf. Section 11.1.2). This means taking the weights as

$$W_j = \left(\frac{1}{B_{1j}} + \frac{1}{B_{0j}}\right)^{-1}, \quad j = 1, \ldots. \tag{A.4.7}$$

When several categories of a determinant of the magnitude of the rate at issue are being compared, the choice of precision-maximizing weights for internal and mutual standardization involves consideration of relative importance among the contrasts. Where such distinctions do not exist, the choice of weights should again reflect the amount of comparative information alone. Such weights, as an extension of those given above, are

$$W_j = \left[\sum_i \left(\frac{1}{B_{ij}}\right)\right]^{-1}, \quad j = 1, \ldots. \tag{A.4.8}$$

APPENDIX 5

Census vs. Case-Referent Approach

Relative Informativeness

Suppose that the study base embodies the rates

$$r_i = \frac{c_i}{B_i}, \quad i = 1, \ldots \tag{A.5.1}$$

for different categories of a determinant of interest (with c denoting the empirical number of cases and B the size of the corresponding base; cf. Section 4.1). Potential interest in these rates, with the base itself as the technical referent (Section 1.6), is mainly of two types:

1. It may be particularistic to the point where the actual realizations r_i are of interest per se.
2. The base experience may be viewed as a (simple random) sample of an infinite amount of experience of its kind, actual or hypothetical, and the