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THE ECONOMETRICS OF FAMILY BUDGETS

By H. S. HOUTHAKKER

University of Cambridge, Department of Applied Economics

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Family budgets have been frequently discussed by the Society or in its Journal, and it is therefore unnecessary to stress here the great contribution they can make to our knowledge of the basic facts of economics. In recent years there have been Mr. Massey's report on the survey of certain groups of middle-class households which he directed in 1938–39 (Massey, 1942), Mrs. Ross's detailed calculations on the clothing consumption of a sample of families from the 1937–38 working-class inquiry (Ross, 1948), and Mr. Nicholson's more comprehensive researches into the expenditure of another group of families from that inquiry (Nicholson, 1949). Elsewhere Professor A. M. Henderson (1949–50) has given estimates of the "cost of children" based on both the pre-war surveys. The Ministry of Labour, which at first had published only a few overall averages for their 1937–38 survey (Ministry of Labour, 1940, 1941), recently made available a classification of expenditures by region and by total expenditure (Ministry of Labour, 1949). Stuvel and James (1950) investigated food expenditure in Dutch family budgets.

When early in 1950 an investigation into the current demand for foodstuffs was started at the Department of Applied Economics it was soon realized that the above publications, valuable though they were in themselves, did not exhaust the wide range of information which the 1937–38 working-class and the 1938–39 middle-class household surveys might be expected to yield. It was therefore decided to undertake a new analysis of the largest possible number of individual returns, transcripts of which the Ministry of Labour (acting also as custodian of the middle-class budgets) kindly put at the Department's disposal. In fact data for about 2,200 working-class households, all of whom supplied clothing information for a whole year, and for the entire sample of middle-class families analysed by Massey were available.* Practically all the information on these returns was punched on cards, and through the generous assistance of a number of organizations with Hollerith equipment (mainly Government Departments) the extensive task of tabulating the figures was made possible. Specific acknowledgments are given at the end of this paper.

The results of this work will be published in a forthcoming monograph of the Department of Applied Economics. The present paper should be regarded as a preface to the latter, setting out the intentions and methods of the analysis, illustrated where possible with numerical examples and dealing with some more general questions encountered in the investigation of household budgets and its application to specific economic problems. The subjects discussed will be found to be largely the same as in the pioneering monograph of Allen and Bowley (1935), which is still without rival as an introduction to the theory and econometrics of this field, though naturally opinions have developed since its appearance and many of its conclusions can no longer be upheld.

* The number of working-class budgets was slightly less than that dealt with in Chapter VI of (Ministry of Labour, 1949) as some forms were unusable. On the other hand, there was one more middle-class budget (making 1,361 families in all) than the number with which Massey worked.

Apart from some incidental remarks we shall not discuss the design of household surveys, but only the analysis of the results, especially the econometric analysis which is intended to isolate and measure the regularities of an economic nature by statistical methods. When regularities belong to economics and when they do not is a delicate point which may be evaded by quoting the convenient definition that "economics is what economists do". This will exclude most work on nutrition, the other main source of interest in family budgets. The application of family budgets to the construction of index numbers and to calculations on the burden of taxation cannot be discussed here either, though some of our remarks will be relevant to these subjects.

1. *The Theoretical Background*

1.0. According to economic theory the consumption of an individual will be determined by his income and the prices ruling on the market in conjunction with his preferences. This will incidentally apply to households only if a certain unanimity between the several persons normally constituting a household is postulated (cf. Samuelson (1950), pp. 374–5).

1.1. *Income* is of course the most obvious cause of differences between the consumptions of different families, and indeed the central factor in all budget analyses. In practice it is often difficult to ascertain, and in any case its influence may be lagged (cf. Tobin (1950), pp. 115–7), so that one may have to work with total expenditure instead, as is done in the present analysis. The gain in statistical precision probably outweighs any theoretical difficulties this may cause; it might even be argued that total expenditure fits much better into a theoretical scheme which effectively ignores savings. Some calculations on the relation between income and total expenditure are given in section 8 below.

1.2.0. One of the assumptions commonly used in estimating the effect of income on consumption is that *prices* are the same for all families in a survey. This may be substantially true for most commodities, but for some items, particularly rent, it is frequently not true; because of income effects this may significantly affect all other items of expenditure (cf. Section 6). Differences in rent are probably an important cause of geographical variations in consumption, especially between localities of different sizes, but in practice they need not always show up because of randomization. Only where the level of rents is correlated with the level of incomes may these regional differences become a source of bias.

1.2.1. Other differences in prices arise when consumers can obtain reductions for large purchases (e.g., block tariffs for gas and electricity); it will be very difficult to allow for this. Falling average prices per unit under two-part tariffs are another matter, however, since only the fixed charge and the marginal price are relevant (cf. Houthakker (1951), p. 360).

1.2.2. Apparent differences in prices may occur if goods are available in a variety of qualities, but this difficulty results only from the necessarily incomplete classification of goods, and does not violate the hypothesis of equal prices for the same item of consumption. It points to an interesting subject of inquiry (cf. Section 4 below).

1.3.0. Variations in *preferences* will also have to be taken into account if one wants to establish a justifiable method of drawing general conclusions from a sample of families; differences in income and prices will never be sufficient by themselves to explain the wide scatter that is found. Such variations should not only be studied because they may bias the estimates of income effects, for they also raise some remarkable problems of their own. The most tractable variations are naturally those that can be associated with observable exogenous factors, while the remainder must be assumed to lead to a probability distribution of some form.

1.3.1. The effect of variations in tastes can only be observed indirectly, viz., from consumption, on which they have a twofold effect. If, for instance, a man decides to keep a cat (*ceteris paribus*) his demand for milk will increase (a *specific effect*), but in order to pay for this he will have to spend less on other goods, which will therefore experience an *income effect*. This distinction is analogous to that between substitution effects and income effects expressed in the Slutsky-Hicks equation (Hicks, 1939, p. 309), which has recently also been interpreted in terms of changes in tastes (Ichimura, 1950–51; Hicks, 1950–51).

1.3.2. The demand for food, clothing, education, etc., evidently depends on the age and sex composition of a family, and since these items have a large share in total expenditure, the resulting specific effects will be accompanied by considerable income effects. Here we have in fact the

most important form of variations in preferences. It should be noted that the specific and the income effects work in opposite directions (except in the case of inferior commodities), for at constant income their algebraic sum over all goods (in money terms) must be zero. Thus a large family which would like to spend more on clothing than a smaller one with the same income may in fact have to spend less because it also has to pay for more food. From this it can further be seen that the total effect of differences in family composition on the consumption of a good will in general depend on income, for whereas the specific effect may remain the same, the income effect is spread out over a larger range of commodities if income is high. A more detailed discussion of the family size problem will be found in Section 5.

1.3.3. The occupations of the members of the household will also influence the demand for various goods. In so far as this influence is of a technical nature (e.g., the fact that heavy manual labour will require much food) it is questionable whether this can be described as a variation in preferences at all; indeed the same thing might be asked about the social conventions which cause middle and working-class households to consume different amounts even though their incomes and other circumstances are the same. The difficulty is still more apparent where relatively many members of the family are at work so that they have to hire domestic help or buy more prepared food. For our present purpose it will be difficult to do anything but describe these differences to varying preferences, though an analysis embracing both consumption and employment might explain them as ultimately due to different abilities and opportunities. In the same way regional variations in consumption patterns (if not due to differences in prices, cf. 1.2.0) may perhaps also be reduced to differences of circumstances.

1.3.4. Family size, occupation and location are no doubt the three main factors on the preference side which make for differences in consumption. We have said already that any other factors will in practice have to be lumped together in a probability distribution of the residuals from some appropriate regression. The question then arises what form this distribution will have. It would be convenient to have a homoscedastic normal distribution, and Allen and Bowley (1935, pp. 140–1) have given reasons why this might in fact be found, but their argument is not convincing since they ignore the fact that consumption cannot be negative (cf. 3.3). It seems probable, if one takes this fact into account, that the distribution is skew (with the median smaller than the mean), and that the variance will increase when income rises as there will then be more scope for variations of taste, income effects being less serious (cf. 3.5).

2. *Objectives of the Analysis*

2.0. The theoretical considerations just outlined provide general directives for the new analysis of the budgets described in the introduction. Our principal intention was to get reliable estimates of the effect of income on consumption, for which purpose the sources of bias we have described had to be eliminated as far as the basic data would allow.

2.1. One classification was available at once, viz. between working-class households (where the head was a non-agricultural worker earning less than £250 a year) and middle-class households (where the head was a civil servant, local government official or teacher earning more than £250 a year). The design of the middle-class inquiry was aimed at comparability with the working-class survey: both referred to four weeks at quarterly intervals during the year (except for clothing, for which a number of families supplied information during 48 weeks), the periods during which they were held overlapped, and the commodity classification was substantially, but not completely, the same (cf. also Henderson, 1949). The presentation of the results in (Massey, 1942) and (Ministry of Labour, 1949) was not consistent, however, and in any case insufficient, as we shall explain. As the detection of social class differences in consumption (cf. 1.3.3) was one of the objectives of our inquiry a uniform treatment of the two groups of budgets was necessary.

2.2. In Mr. Massey's paper the families were classified by the yearly income of the head as stated during the first week of the inquiry. No income figures for the working-class households were available, so this criterion could not be adopted. In Chapter VI of (Ministry of Labour, 1949), which refers to the working-class budgets here analysed, the basis of classification was average total expenditure during the four weeks of the inquiry, except for clothing, where the average over 48 weeks was used. The latter correction could not be applied to the middle-class

budgets, only part of which contained continuous clothing information. The reason for the Ministry's adjustment was no doubt the erratic behaviour of clothing purchases from week to week, but this occurs also in many other items, and the chance that these random deviations (which will be negatively correlated) cancel out is obviously greater if no separate adjustments are made. The criterion finally adopted was *expenditure on all items during the four weeks of the inquiry*.

2.3.0. A classification by total expenditure only is in general not sufficient to remove the disturbances mentioned in 1.3 above. It is usually found that family size is positively correlated with total expenditure, at any rate within one social class, the reason being that large families usually contain several earners. The two inquiries under consideration are both striking illustrations of this tendency, and attempts to calculate income elasticities from the figures in Chapter VI of (Ministry of Labour, 1949) are therefore doomed to failure. The same applies to the classification by income of the head of the household used by Massey (1942); in that group of the population income rises with age and so does, broadly speaking, the number of children and their age. *Hence it is essential to classify families by income (or total expenditure) and family size simultaneously;* because of the correlation just indicated two one-way classifications will not suffice.

2.3.1. The conclusion in italics will seem trivial to those with experience in family budget analysis; nevertheless it is emphasized in view of current preparations for a new budget survey in Britain. It is much better to have a sample of manageable size that can be properly analysed (about 5,000 should be ample to get reasonable cell-frequencies), than one which is so large and costly as regards fieldwork that only part of the available information can be extracted. The contribution of a second lot of 5,000 budgets to the precision of an over-all average would be negligible; it could only be justified by the additional cross-classifications it makes possible.

2.3.2. A two-way classification as required was used by Nicholson (1949) and Henderson (1949, 1950), but not for detailed items of expenditure. Moreover, both these authors were especially interested in the effect of children on consumption, and consequently confined themselves to a much smaller number of budgets than could be investigated here. Some calculations on the material used by Nicholson are given in Table 1.

2.3.3. The simplest basis for grouping families by their composition is the number of persons, which has been used here. It has the advantage of being determinate in advance, unlike "family types" or the number of "unit consumers" (cf. Section 5), although the latter may give more precise groupings (i.e., a smaller variance within groups) after they have been agreed on.

2.4. Regional variations are probably a less serious source of bias than family size; in any case a complete analysis of this factor was impossible, because the middle-class budgets did not indicate the location of the household. In the punching of the working-class budgets the possibility of regional analyses was taken into account, but in this paper we shall only present some calculations on differences due to living in London (cf. Section 6).

2.5. We have already mentioned the problem of qualities (1.2.2)—a very important one for the combination of time series and family budgets in demand analysis (cf. Tobin (1950), especially pp. 147–8), as the former usually refer to some kind of physical measure and the latter usually to money expenditure. Since changes in income and family size will lead to shifts between more and less expensive qualities of a good the behaviour of quantity and expenditure will diverge. For the working-class households quantity figures in respect of a number of commodities, mainly foodstuffs, were available on the documents. They were not always complete, however, and had not been analysed previously. Special precautions in the tabulation of the punched cards made it possible to calculate the average prices per unit paid in the various income-size groups. This information proved to be of great interest, not only because of its relevance to demand analysis, but also because of its possible application to the problem of family size influences (cf. 5.4.3).

2.7. At the time when the analysis was started it was also believed that the intercorrelations between different items of expenditure, after correction for income, etc. (cf. Allen and Bowley (1935), pp. 89–96), might lead to estimates of the substitution effects between them, thus providing estimates of cross-price elasticities where time series methods failed. Further theoretical research revealed, however, that although these intercorrelations are closely related to the cross-price elasticities, the mathematical link between them is such that no useful estimates can be obtained.

2.8. To sum up, the main features of our analysis are:

a. Two-way classification by total expenditure and by number of persons in the household.

b. Uniform treatment of working and middle-class households, with subsequent comparisons between them.

c. An investigation of average prices, with application to family size problems.

Moreover the tables in the monograph will show not only average expenditure by each group of families, but also the number of households who reported having bought something. The commodity classification will be as detailed as possible.

3. *Engel Curves*

3.0. The investigation of the relation between income (or total expenditure) and the expenditure on particular items or groups of items, represented by so-called Engel curves, is still the most important function of family budgets in econometrics. In practice its discussion cannot be divorced from the questions of family composition, social differences and regional variations that will be dealt with below, nor would it be advisable to discuss these questions first; some anticipations of later remarks are therefore inevitable. We do not propose to speak here of the interpretation of Engel curves as short or long run phenomena and similar problems; our topic is how to derive meaningful, accurate and comprehensive descriptions of the regularities that are apparent in the data.

3.1. It will be remembered that in Allen and Bowley's admirable *Family Expenditure* (1935) most of the income-expenditure relations investigated were found to approximate closely to straight lines. The same technique was applied by Allen (1942) to data for different "types" of families from the U.S. survey of 1935–6,* and by Henderson (1949) to British data; in both cases different straight lines were fitted for different family types. Nicholson (1949) introduced quadratic terms into the regression equations because of the curvilinearity which became evident when families were grouped both by total expenditure and by size.

3.2. These linear and quadratic functions (and more generally all polynomials) have the useful property, unlike other mathematical approximations, that the regressions for individual commodities add up identically to total expenditure (Nicholson (1949), pp. 388–9). On the other hand, the expenditures which they predict will be negative for some part of the positive income range, a defect that is not shared by some functions, such as the logarithmic ($\log y = a + b \log x$), that are excluded by the additivity condition. Another drawback of polynomial regression is that, unless the range of observation is much larger than is usual in budget surveys, the sampling covariance of the regression coefficients is frequently very high as a variable and its square will then be highly intercorrelated.

3.3. It might be thought that theoretical considerations would support polynomial regression because of its additivity. Thus Allen and Bowley (1935, pp. 135–7) have discussed how linear Engel curves fit into a simple but by no means trivial special case of consumption theory.† Unfortunately their argument cannot be accepted for the reason already mentioned, viz., that the essential non-negativity of consumption is ignored. As soon as the appropriate boundary conditions are introduced Engel curves turn out to be much more complicated functions of income, which show kinks at points where new goods enter the budget or others drop out.‡ It is a matter of theoretical indifference what analytical approximations are applied to the resulting theoretical Engel curves, which are continuous but not everywhere differentiable. Further difficulties are connected with the problem of allowing for random variations in preferences under these circumstances (it is one of the most valuable contributions of *Family Expenditure* to have introduced this problem into consumption theory). Nevertheless it may be expected that for most commodities, especially foodstuffs, the slope of the Engel curves will diminish as income rises, as room has to be made for goods that are only consumed after a certain income has been reached.

* The seven family types distinguished in that survey were based on the number of children and the number of adults.

† It appears, however, that quadratic Engel curves (or higher polynomials) cannot be derived from any explicit utility function.

‡ This will be more fully explained in Houthakker (1952).

3.4. Strictly speaking there is no reason why the same type of function should be applicable to all commodities, but this is of course highly convenient from a computational point of view. In practice it is also desirable to use expressions that are amenable to maximum-likelihood estimation; graphical methods are too unreliable for anything but exploration, particularly when the observations have different weights. This rules out the formula suggested by Tornquist (1940), where the expenditure on an item is the ratio of two linear functions of total expenditure.

3.5. Statistical considerations should also have some influence on the choice of the regression function. In ordinary regression theory the residuals are assumed to be normally and independently distributed with constant variance. We have already expressed doubts on the normality and homoscedasticity of the expenditures (cf. 1.3.4); it seems that a logarithmic transformation might provide for these difficulties, for if the logarithms are assumed to be normal and homoscedastic the original distribution will be skew with variance proportional to the mean. As to independence, this has also been questioned, notably by Duesenberry (1949), who stresses the interdependence of preferences. No statistical methods for this problem have yet been developed (they would probably amount to a generalization of serial correlation techniques to two or more dimensions), and we therefore prefer to ignore it for the time being.

3.6. In the regression functions income (or total expenditure) will not be the only variable; family size, etc., will also have to be included. This again has some bearing on the shape of the Engel curve; it was found, for instance, that a social class factor cannot easily be combined with a hyperbolic Engel curve.

3.7. The actual choice of the type of function is clearly a complicated matter, which will have to be mainly decided by experiments on the data. My colleague, Mr. S. J. Prais, intends to report on his investigations into this subject separately. The evidence seems to support logarithmic Engel curves, as used in Table 1 below and elsewhere. These have the additional advantage of agreeing with the logarithmic demand functions favoured by demand analysts, e.g., by Stone (1945, 1948, 1951).

3.8. These logarithmic functions,

$$e_i = d_i e^{\beta_i},$$

where e_i is the expenditure on the i^{th} commodity and e total expenditure, do not strictly fulfil the additivity condition $\Sigma_i e_i = e$. Nevertheless it will be realized that the difference

$$e - \Sigma_i \alpha_i e^{\beta_i} = e(1 - \Sigma_i \alpha_i e^{\beta_i - 1})$$

may be close to zero for a considerable range of values of e, since the regression functions are fitted to observations which themselves satisfy the additivity condition.

4. *Qualities*

4.0. The statistical information on average prices per unit for a number of commodities and countries will be surveyed in a forthcoming paper (Houthakker and Prais, 1952). Here we shall only give a brief review of the principal points inasmuch as they affect the British surveys.

4.1. The analysis of qualities on the basis of family budgets starts from the assumption (discussed in 1.2) that all families can buy at the same prices, and that therefore if they do pay different amounts per unit for the same good the qualities bought must be different. "Quality" here includes service in the shop, etc. The problem then arises how qualities, thus defined, vary with income, family size and other determinants of consumption.

4.2. The necessary data on quantities consumed were only available for the working-class budgets, covering about 40 commodities. It was found that in several cases households had not reported quantities, although they had spent something on the item concerned; this made it difficult to analyse quantities by themselves. By excluding such families* it was nevertheless possible to compute average prices for each of the total expenditure-family size groups in which families were classified.† Distinct regularities emerged when average prices were plotted

* Except in one or two commodities they never amounted to more than a few per cent. of the total number of families who reported expenditures.

† Broadly similar calculations were made by Mrs. Ross (1948) for clothing in a small sample of the working-class budgets analysed here. The detailed data she worked with could not be used for the present inquiry.

against total expenditure, though naturally the correlation varied from commodity to commodity. In general average prices rise with income, but large families spend less per unit than small ones. A clear example of these tendencies in the case of tea is given in Fig. 1 (the symbols do not always represent the same number of families: the weights are usually largest in the 4-person households and tend to decrease from left to right; groups with less than 5 families were omitted).

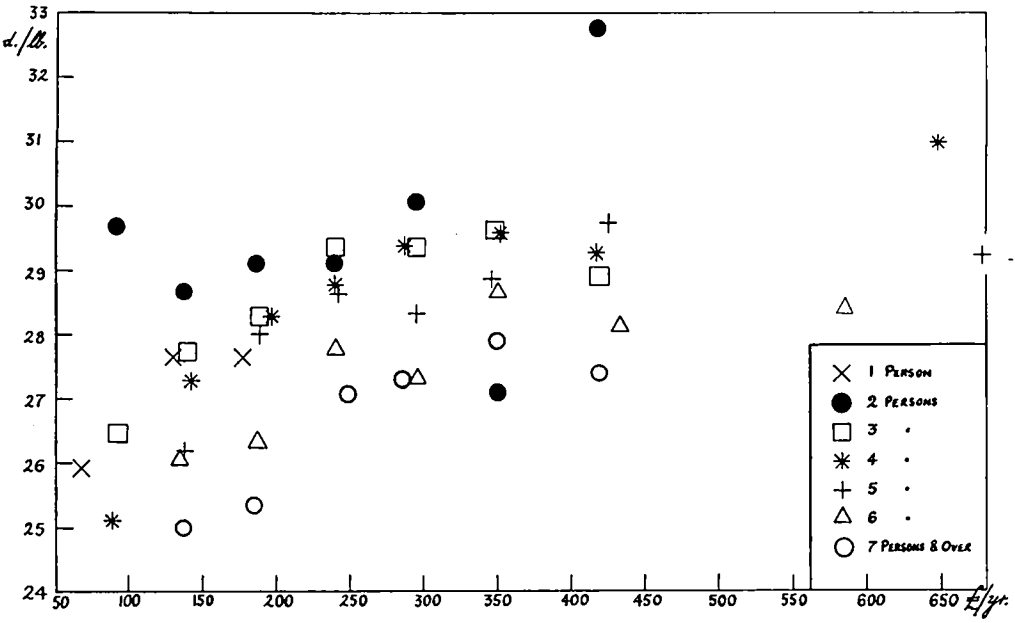

FIG. 1.

4.3. In order to simplify the tendencies evident in Fig. 1 (which is typical of many other commodities) it is necessary first to eliminate family size. The simplest method of doing so is to divide total expenditure by the number of persons; the curvilinearity could be avoided by taking the logarithm of total expenditure per head. The result is shown in Fig. 2 for tea; Fig. 3 is a similar diagram for bacon and ham. It will be seen that apart from the 1-person households, which are a small and special category in any case, the following relation holds fairly closely,

$$v_i = a_i + b_i \log e/n,$$

where v_i = average price per unit of the i^{th} good, e = total expenditure, n = the number of persons and a_i and b_i are constants. The data for different family sizes no longer show any systematic pattern, which proves that family size has been largely eliminated. This observation applies to a considerable number of commodities and will prove to have important consequences (cf. 5.4.3). Some estimates of a_i and b_i are (with standard errors in brackets and correlation coefficients):

	a_i	b_i	r^2
Sugar	2·18	·258 (±·040)	·44
Bacon and ham	7·88	5·36 (±·56)	·64
Beef and veal (home prod.)	3·32	5·95 (±·91)	·45
Pork	6·49	5·54 (±·119)	·31
Tea	20·5	4·42 (±·46)	·65

Prices are in pence per lb. and total expenditure in £ per year.

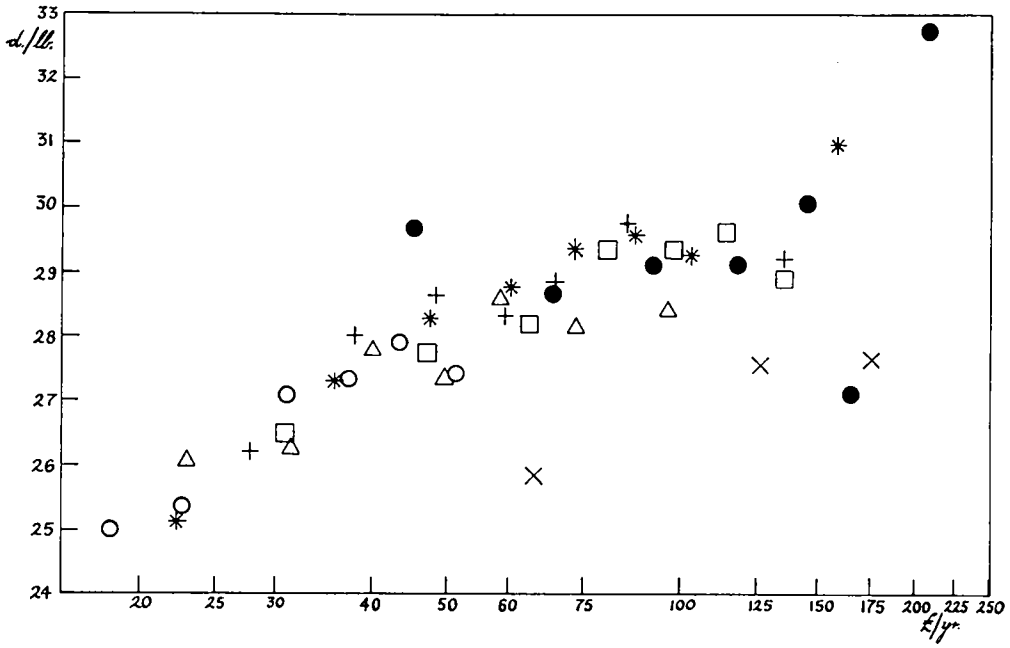
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FIG. 2.

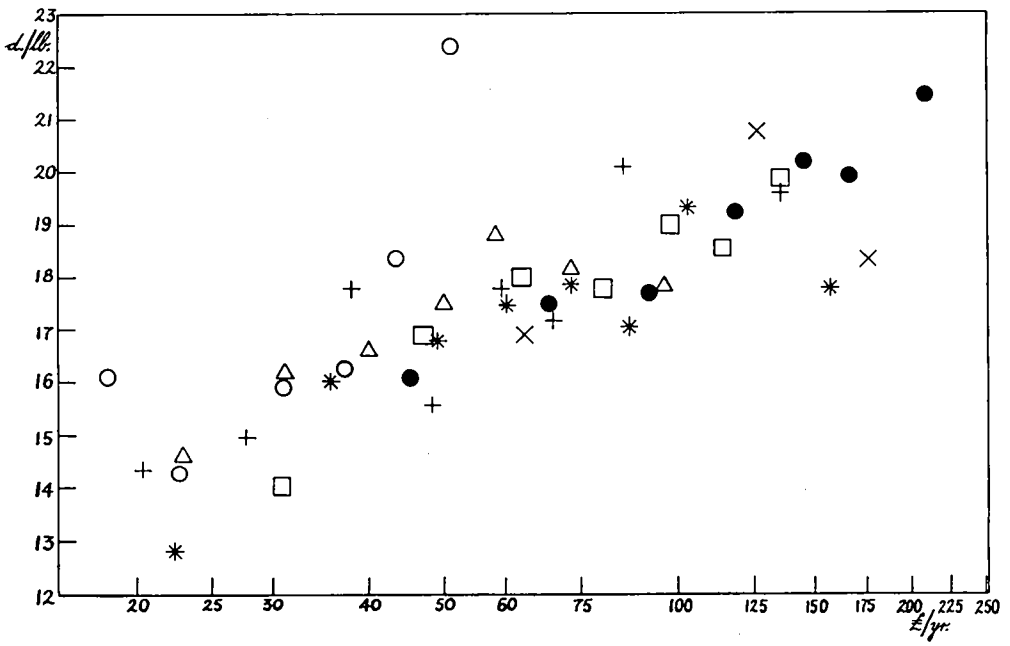
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FIG. 3.

4.4. From formulae of this kind one can deduce "quality elasticities", defined as the proportionate rise in the average price per unit associated with a small proportionate rise in income. In our case this is

$$E_i = \frac{e/n}{p_i} \frac{p_i}{\delta e/n}.$$

For the above equations one finds (at $e/n = 50$, as E_i is not constant) for

Sugar	·043
Bacon and ham	·160
Beef and veal (home prod.)	·192
Pork	·151
Tea	·069

These quality elasticities are the differences between the expenditure elasticities usually provided by family budgets and the quantity elasticities (referring to physical weight or to value at constant prices) needed for demand analysis by time series methods.

5. *Family Composition; Unit Consumer Scales*

5.0. The distinction between specific and income effects of variations in preferences, introduced in 1.3.1 above, is crucial for the problem of the influence of family composition on consumption. Family consumption in quantitative term means a specification of the members of a household by age and sex, and the problem is how to relate the consumption of different items to this specification. It is evident that differences in family composition do not affect all items of expenditure in the same way. Thus the birth of a baby will have no specific effect on the household's demand for men's clothing, but a considerable specific effect on its demand for babies' clothing. This is an extreme example, but more moderate differences in specific effects will be found between nearly all commodities. On the other hand, the income effect is by its nature relevant to all goods, with an intensity dependent on their respective income elasticities.

5.1.0. The usual way of taking family composition into account is by so-called "equivalent adult" or "unit consumer"* scales, which reduce the composition vector to a weighted sum, the number of unit consumers. These scales usually apply to total food consumption, in which case the weights are allegedly based on physiological considerations; sometimes also to clothing or housing. Their economic significance has been questioned, especially by Allen (1942), who in fact interpreted them much more charitably than most constructors of these scales intended. Various points in his discussion call for some comment.

5.1.1. Professor Allen observes that the unit consumer scales will be different according to the item concerned, and that in addition there will be some average scale for reducing total expenditure. We may associate the first type with the specific effects, and the second type with the income effects distinguished above. If the members of a family are divided into m categories and the number of persons in the i^{th} category is denoted by n_i, the number of unit consumers for the j^{th} commodity ($j = 1, \ldots g$) will be $(k_{j1}n_1 + k_{j2}n_2 + \ldots + k_{jm}n_m)$, where the k_{ji} are the weights. Similarly the number of unit consumers on the total expenditure scale may be expressed as $(k_1n_1 + \ldots + k_mn_m)$. In each case the first k is assumed to be unity to avoid proportionality (all other categories of persons are thus given a weight relative to the first category; if this is adult males the name "equivalent adult (male)" is explained).

5.1.2. The Engel curves considered by Allen are straight lines†; the expenditure function becomes

$$\frac{e_j}{\sum_i k_{ji}n_i} = a_j + b_j \frac{e}{\sum_i k_in_i} \qquad (j = 1, \ldots, g) \qquad \ldots \quad (1)$$

* This term was suggested by Mr. M. J. Farrell, and will be used here throughout as it seems preferable both on logical and on linguistic grounds.

† The objections discussed in 3.2 and 3.3 are not relevant if one confines oneself to a section of the income range where the set of goods actually bought remains unchanged.

in the notation of 3.8; a_j and b are constants and all summations cover the whole range of the suffix concerned. After some multiplication and summing over all goods we get

$$\sum_j e_j = \sum_j \sum_i a_j k_{ji} n_i + \sum_j \sum_i b_j k_{ji} n_i \frac{e}{\sum_i k_i n_i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since $\Sigma e_j = e$ this implies the following identities for all e and n_i in the range considered:

$$\sum_i \sum b_j k_{ji} n_i \equiv \sum_i k_i n_i \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\sum_j \sum_i a_j k_{ji} n_i \equiv 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(3) leads to

$$b_1 k_{1i} + b_2 k_{2i} + \ . \ . \ . \ + b_g k_{gi} = k_i \qquad (i = 1, \ . \ . \ . \ . \ , m), \quad . \quad (5)$$

i.e., the total expenditure scale is an average of the specific scales weighted by the income derivatives. If we take $i = 1$, for which all the k's are one, we find $\sum_i b_i = 1$. The other identity (4) defines a system of homogeneous equations

$$a_1 k_{1i} + a_2 k_{2i} + \ . \ . \ . \ + a_g k_{gi} = 0 \qquad (i = 1, \ . \ . \ . \ . \ , m). \quad . \quad . \quad (6)$$

We may assume that there are generally fewer categories of persons than items of expenditure ($m < g$), in which case the conditions (6), regarded as equations in the a's, can normally be fulfilled. If there are more types of persons than goods, however, these scales can only hold if the scales are linearly dependent or if all the a's vanish.

5.1.3. The doubts voiced by Allen (1942) on the validity of these scales are based on an interpretation of the linear Engel curves fitted separately for different family types without actually using such scales. He predicted the behaviour of the slopes and intercepts of these lines for various commodities by *a priori* arguments, and then found some discrepancies between his theory and the estimated slopes and intercepts. Unfortunately no significance tests were made (in any case the estimates were obtained by graphical methods), which is all the more regrettable because the linearity of the functions is not confirmed by an inspection of the data. This makes graphical estimation very unreliable, as is in fact borne out by a least-squares analysis of some of the figures Allen used. Even if the standard errors were as small as 10 per cent. of the regression coefficients to which they belong, most of his conclusions would be well below the usual levels of significance. The interpretation of the estimates thus becomes largely a matter of judgment, and one may well find the agreements between theory and facts, particularly in the case of food, more striking than the disagreements.

5.1.4. One of Allen's objections to equivalence scales was that the effect of an additional child on food expenditure appears to depend on how many children there are already. This difficulty could easily be removed by suitably extending the concept of a "category of persons" (cf. 5.3.1), but a more elegant explanation, based on the distinction between specific and income effects, turns the apparent discrepancy into a confirmation of the theory. Consider equation (2), leaving out the summation over commodities, and differentiate with respect to n_h.

$$\frac{\partial e_j}{\partial n_h} = a_j k_{jh} + b_j e \frac{(\sum k_i n_i) k_{jh} - (\sum k_{ji} n_i) k_h}{(\sum_i k_i n_i)^2}. \quad . \quad . \quad . \quad . \quad (7)$$

Generally speaking the net effect of an additional child on food consumption will increase with total expenditure, and the numerator of the coefficient of e will therefore be positive if b_j is positive (as is no doubt the case with total food). This numerator does not depend on n_h, since the terms with n_h cancel out. Differentiating again we get

$$\frac{\partial^2 e_j}{\partial n_h^2} = - \ 2 b_j e (k_{jh} \ \Sigma k_i n_i - k_h \Sigma k_{ji} n_i) \frac{k_h}{(\Sigma k_i n_i)^3}. \quad . \quad . \quad . \quad . \quad (8)$$

and k_h being positive this implies

$$\frac{\partial^2 e_j}{\partial n_h^2} = - \ 2 \frac{\partial^2 e_j}{\partial n_h \partial e} \ \frac{k_h e}{\Sigma k_i n_i} < 0. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Consequently the net effect of an additional child will be smaller the more children there are already, as noted empirically by Allen and by Nicholson (1949, p. 388). It is indeed intuitively obvious that under the above conditions an increase in the number of children, which will reduce total expenditure per unit consumer, must lead to a reduction in the food consumption of the initial members of the family, and the value of this reduction will increase with the size of the family. In other words the income effect will counteract the specific effect to an increasing extent. We should add, however, that the decreasing net effect of children observed in the data may also be due to economies of scale.

5.2.0. The principal attraction of unit consumer scales, when improved by distinguishing specific and income scales, is that they permit all households to be dealt with on the same footing. This is not the case if one divides families into types and analyses these separately, as was done by Allen (1942) and Henderson (1949, 1950), and also by Nicholson (1949) for a selection from the U.K. working-class survey. Nicholson's investigation was an important advance in the study of household consumption, but at the same time clearly illustrates the limitations of this procedure.* In the first place the sample had to be selected very carefully, excluding all families whose composition had changed during the survey (e.g., where a child was born), and where there were persons aged 14 and over other than one man and one woman. This was necessary as families were to be classified by the number of children, but it reduced the number of available budgets from about 2,000 to about 800. In addition London households were kept separate, as their consumption was thought to be too much affected by higher rent and travelling expenses; this further reduced the number of families to 704. Some of the cell-frequencies consequently became quite small, although for regression purposes this does not matter if cell-averages are weighted by the number of observations they represent.

5.2.1. The ages of the children were not taken into account in Mr. Nicholson's calculations, which may introduce a slight bias into the regression coefficients for total expenditure, because the average age of the children within each family type will rise with the age of the father and hence, in many cases, with his earnings. Moreover a family with two small children is put in the same category as one with two children near school-leaving age, although the consumption pattern of the former family may be more like that of a family with only one older child; in a more general form this is of course the principal argument against all family type classifications. The separation of London families is also a questionable point; are they really more different from Midland families than Midland families are from those further North? One wonders if it is wise to go so far in sacrificing coverage and size of sample to an unattainable uniformity of the observations; it may be better to take a greater variety, and rely to some extent on randomization.

5.2.2. It might be objected that in our own analysis we also classify families by size (cf. 2.3.3) so that we are also working with family types, and possibly not even very suitable ones as such. The difference with the authors just mentioned is that our classification is used only to get sufficient variation in the independent variables; in the actual regression analysis all families are dealt with simultaneously. An example of this referring to the Nicholson sample is given in Table 1.

5.3.0. In order to substantiate the favourable view of unit consumer scales here taken it is necessary to extend their scope in various ways. To begin with we must consider them in connection with non-linear Engel curves. Formula (1) in 5.1.2 should then be rewritten as

$$\frac{e_j}{\Sigma k_{ji} n_i} = a_j\left(\frac{e}{\Sigma k_i n_i}\right) + b_j\left(\frac{e}{\Sigma k_i n_i}\right)\frac{e}{\Sigma k_i n_i}, \quad . \quad . \quad . \quad . \quad (10)$$

where a_j and b_j now depend on total expenditure (or income) per unit consumer. This generalization clearly covers all possible Engel curves. The conditions (5) and (6) will then no longer hold identically, except for $i = 1$. More particularly, if the k_{ji}'s are constants the k_i's, as given by (5), will in general vary with total expenditure. Thus if a category of persons has a high relative weight in the specific scales for items with a low income elasticity, and conversely, then its relative weight in the income scale will decrease as income rises. Concerning the conditions (6), one

* No criticism is intended or implied, since for an analysis with limited resources and with special emphasis on children the family type method may well be appropriate, especially if one regards Allen's criticism of equivalence scales as decisive.

may perhaps rely on the statistical fitting procedure, as explained in 3.8. The proof of (9) requires some additional assumptions which are not difficult to work out.

5.3.1. The method of dividing the members of a family into categories provides a further means of improving the applicability of unit consumer scales. The simplest idea is to work with the number of persons in each age-sex group irrespective of the numbers in other groups. If one wants to allow for non-linearities of the kind mentioned in 5.1.4 (which, as we have seen, may not be necessary from a theoretical point of view), it is perfectly possible to introduce "children beyond the second" or "children in families with two or more women" as separate categories (an example is given by Quenouille (1950), p. 29). In this way most interactions and higher-order effects can be taken into account, without abandoning the formally linear unit consumer scales. Similarly one may introduce a constant into the scales by putting one of the n_i equal to unity; this trick, which amounts to interpreting one of the categories of persons as "the household independent of its composition", is necessary for those rare commodities which experience no specific family size effects at all.

5.4.0. Having attempted to restore unit consumer scales to the favour of consumption analysis we must now discuss the actual estimation of the weights. The Engel curves we have in mind can be described by

$$\frac{e_j}{\sum_i k_{ji} n_i} = f_j\left(\frac{e}{\sum_i k_i n_i}\right), \qquad (11)$$

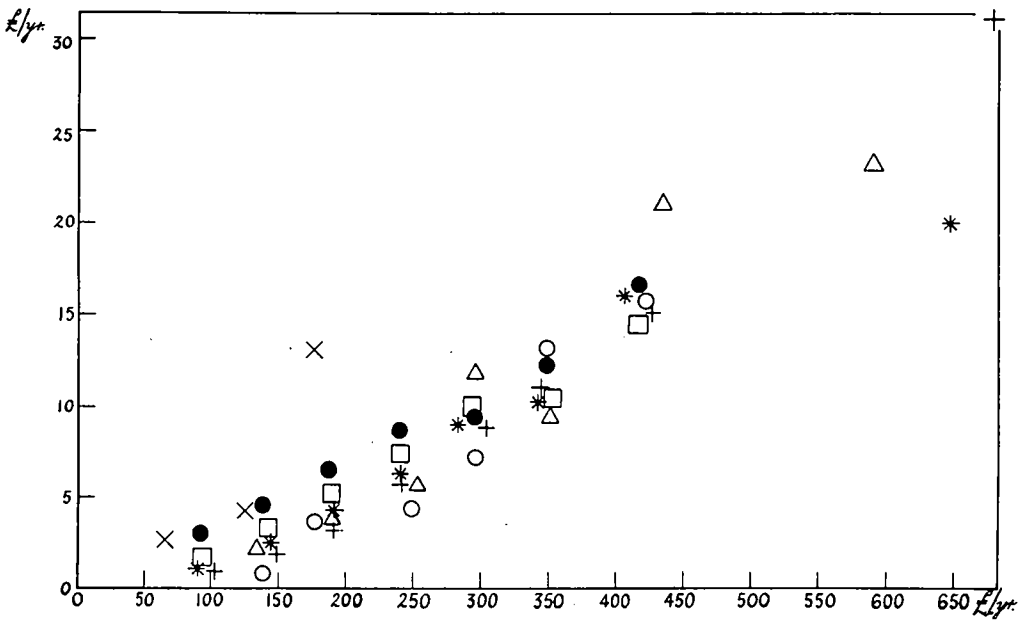
where the k_{ji} are still constant but the k_i are not necessarily so (cf. 5.3.0). It would be convenient if we could proceed with the estimation in two stages, one being aimed at the specific scales and the other at the income scale. Such a procedure would be possible if there were variables for which the specific scale was known from external evidence, for then the left-hand side of (11) would be known and the parameters on the right-hand side could be estimated by maximum likelihood methods once the mathematical form of f_i is specified. The resulting income scale could then be regarded as known for the estimation of the specific scales. The alternative procedure of first estimating all the specific scales and then calculating the income scale by means of formula (5) evidently offers formidable difficulties.

5.4.1. Computationally the possible variability of the income scale is of course a source of great complications in an already very complicated problem. Fortunately one can find reasons for expecting that this variability will not be very important in practice. The principal characteristic of unit consumer scales is the weight of school-children in relation to adults. Now the specific effect of children is mainly felt in food, clothing and education. Of these three it is nearly always found that food is income-inelastic, clothing has an elasticity of about unity and education has a high elasticity. As a first approach one may therefore suppose that in the income scale these three specific effects will balance to a considerable extent, and that one is provisionally justified in working with constant income weights.

5.4.2. Next one has to find items for which the specific scale can be predicted with some confidence. Foodstuffs are clearly unsuitable, since they are normally consumed collectively; we have to look for more personal items, of which clothing is the most promising example (cf., however, 5.4.3). Men's, women's and children's clothing are stated separately in the two surveys under consideration, and for the first two categories the specific scales are quite simple.* For the income scale one may as an approximation take equal weights for all members of a household, merely in order to get a first impression. The original figures for women's clothing, plotted against total expenditure, are depicted in Fig. 4, while Fig. 5 gives expenditure on women's clothing per woman in relation to total expenditure per head. It can be seen that this transformation of the variables largely eliminates differences between families of different size, though the households with 6 and more persons still seem to be somewhat distinct. This is no doubt due to the crudity of the method; if the weights had been determined by regression analysis a closer fit would have been obtained.

5.4.3. This preliminary result may help to show the reality of the relationship expressed in (11). Nevertheless it would be desirable if the income scale could be estimated from a wider

* The only problem is how to weight persons of 14–17 as compared to persons of 18 and over, no further information on ages being available. In Fig. 5 these two categories have equal weights, but this could no doubt be improved on.


£/yr.
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25
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150
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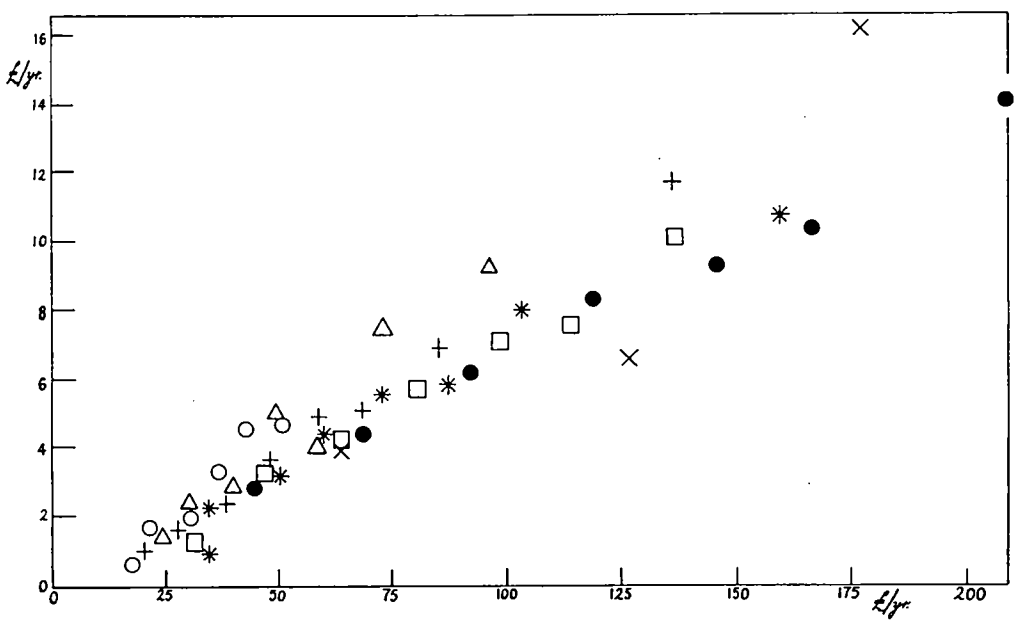
£/yr.

FIG. 4.


£/yr.
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125
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175
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£/yr.

FIG. 5.

range of data. Here the material on average prices paid discussed in Section 4 provides a solution. The quantity of a good consumed will nearly always depend specifically on family size, but in many cases the quality will only be related to this factor through income effects. This will not always be true, not only for the reasons mentioned in 2.1.0 and 2.1.1, but also because a household may buy different kinds of, for instance, fish or vegetables if there are children. Nevertheless for commodities such as tea, meat, flour, etc., the specific effect of family size should be negligible. Figs. 2 and 3 have already shown that even a simple income scale is remarkably successful in eliminating income effects on qualities, so that this type of analysis seems to open great possibilities.

5.4.4. In applying this approach it is of great importance to choose an appropriate type of function, for experience shows that in multiple regression analysis a wrong choice of regression curve may lead to serious biases in the estimates.* In the case of qualities there was considerable evidence for a semi-logarithmic function, which combined with (11) leads to the equation

$$p_i = a_i + b_i \log \frac{e}{\Sigma k_i n_i} + \varepsilon. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In this case the error term ε might well be assumed to be normally and independently distributed (cf. 3.5), but the estimation of the parameters is greatly complicated by non-linearity. Customary computation techniques are powerless here, and the only possibility seems to be an iterative solution using electronic equipment. At the time of writing no results thus obtained are available yet, but it is hoped to give them in the Monograph together with a discussion of the technique of calculation.

5.5.0. The last problem concerning family composition which we have to discuss is the so-called "cost of children". This has recently attracted some attention because of the interest of the Royal Commission on Population in children's allowances and their possible effect on differential fertility. We do not propose to go into the social or ethical aspects of this question, but will only try to indicate briefly the relevance of family budgets to its analysis in numerical terms; some conceptual discussion cannot be avoided, however.

5.5.1. In one of the papers presented to the Royal Commission on Population Hajnal and Henderson (1950) put the question as follows: "How great an addition in income is needed to make parents as well off as a childless couple?" This question, if meaningful at all, is not one on which expenditure data have anything to contribute, since they tell us nothing of the pleasure or displeasure which the presence or absence of children, quite apart from their effect on consumption, will cause to their parents or potential parents. The authors in fact mean something more restricted, viz., "that increase of income which enables the parents to pay any additional expenditure occasioned by a child and then to have the same income left over for their own use". In another paper Henderson (1949–50) has described this as a compensating variation, in analogy with a synonymous concept in the theory of consumer's surplus. It might be added that the latter concept is defined for given and fixed preferences, a condition which is not fulfilled in the present case.† Henderson's approach is based on two assumptions, viz., that expenditure can be allocated between parents and children, and that parents' satisfaction depends only on their personal consumption, if it can be calculated.

5.5.2. The first assumption leads us back to the discussion of unit consumer scales, but this technique is not used by Henderson or by Nicholson (1949), who made similar calculations. They rely instead on the analysis of "standard commodities", introduced by Rothbart (cf. Madge,

* For this reason it is also important to classify the observations in an efficient manner if their number is too large for ungrouped computation (cf. also Nicholson (1949), p. 363). How an optimal classification (which would minimize the covariance matrix of the estimates subject to, for instance, a constraint on the number of groups) can be obtained is incidentally a difficult problem which might repay the attention of statisticians.

† A similar remark applies to the analogy between the two effects of variations in preferences and the Slutsky-Hicks equation (cf. 1.3.1). In our case the analogy seems more legitimate than in Professor Henderson's case: the income effect of taste variations is interpreted in a purely nominal sense, not necessarily related to any change in utility; the extended compensating variation on the contrary seems to have utility implications as well. We should like to stress that without additional assumptions data on observed consumption provide no information whatever about the effect of variations in preferences on the level of utility.

1943), which consists in comparing families with different numbers of children by their expenditure on items solely consumed by the parents, such as drink, tobacco and adult's clothing. For instance, if a family with two adults and no children spends £50 per year on adult clothing out of a total expenditure of £400 per year, and a similar family with one child spends the same on adult clothing out of £500 per year, then the expenditure attributed to the child is £100 per year. This method consists, in our terminology, in finding the income scale for variations in family size from expenditures for which the specific scale is known. We have already seen that quality data may be more effective for this purpose (cf. 5.4.3), but these were not available to Henderson and Nicholson. In any case the usefulness of household budgets for this purely descriptive purpose is incontestable.

5.5.3. By interpreting the figure thus found as the "cost" of a child Henderson gets on more dangerous ground, however. This somewhat commercial term suggests, in accordance with his second (implicit) assumption, that the consumptions of parents and children are independent in the sense that children's consumption is a prior charge determined by their "needs". This assumption would hold if the expenditure function could be expressed as

$$e_j = f_j(e) + g_j(n_c), \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where n_c is the number of children. Such a function was used by Allen and Bowley (1935, p. 18), but it has since been found to be misleading, particularly by Allen (1942), who proved that the first right-hand term in (13) will depend on n_c as well. Henderson's and Nicholson's calculations provide further confirmations of this phenomenon. The practical meaning of this is that if family income rises, part of the increase will be spent on children's consumption, although it is difficult to see in what way the "cost" of a child has risen. Consequently if parents were subsidized to the extent that they bought as much adult's clothing as childless couples, the former category would in fact be overcompensated because they also derive satisfaction from the larger expenditure on children which the subsidy makes possible. The "cost" interpretation appears to be more appropriate to professional foster-parents than to the people whose birth-rate the Royal Commission on Population wanted to influence.

6. *Regional Variations*

6.0. No extensive investigation of regional differences in consumption was contemplated for the present analysis, although for the working-class budgets the possibility of such an investigation at a later date has been kept in mind during the planning of the punched card work.* Some calculations on this subject have been made for the sample studied by Nicholson (1949), where London families were separated from those in the remainder of Great Britain. (Our own sample also covered Northern Ireland.)

6.1. All households in Nicholson's sample contained one man, one woman and 0, 1, 2 or 3 children under 14 years. In order to deal with all of them uniformly an expenditure function had to be used in which the number of children was the only family size index, as their ages were unknown and no adult/children weighting could be used. The expenditure of London families was assumed to differ by a constant percentage from the expenditure of similar families elsewhere. The regression formula used was

$$\log e_j = a_j + b_{1j} \log e + b_{2j} n_c + b_{3j} l, \quad . \quad . \quad . \quad . \quad (14)$$

where b_{1j} is the elasticity of the j^{th} item of expenditure with respect to total expenditure, b_{2j} is the children coefficient, n_c is the number of children, b_{3j} is the London coefficient, and l is a dummy variable equalling 1 if the family lives in London and 0 otherwise. The meaning of b_{2j} and b_{1j} can be seen more clearly if we consider the effect of a change of one unit in n_c or l on e_j; this works out at 100 $(10^{b_{ij}} - 1)$ per cent. of e_j $(i = 2, 3)$. The percentage differences thus obtained are also given in Table 1; we see, e.g., that an additional child increases expenditure on butter by 6·1 per cent. on the average, but that the mean expenditure on butter of families in London is 31·2 per cent. below that of comparable families elsewhere.

* In general the cards are intended to be suitable for almost all conceivable analyses of the data without further reference to the original documents.

6.2. Inspection of Table 1 shows that the standard errors of the total expenditure elasticities are on the whole fairly small. Only one commodity is found to be significantly inferior, viz., condensed milk; the negative elasticities of three other goods have large standard errors. The effect of children is significantly positive for most food items and for clothing and education, but most other commodities experience only income effects so that the children coefficient is negative. It can even be seen that the children coefficient tends to be largest in absolute value for highly income-elastic commodities such as theatres, holidays and food for animals (although this might also indicate a negative specific effect of children). The very large percentage difference attributed to a child in children's clothing reflects the fact that childless households spent very little on this item (what they did buy was presumably intended for children born after the survey). The London difference is clearly dominated by the specific effects on rent and travelling expenses (cf. 1.2.0), which have corresponding negative income effects on most other commodities. Unfortunately there was an imperfection in the data which has tended to exaggerate the London difference in the case of highly income-elastic commodities, e.g., furniture, carpets and education. If the number of families in a group was small no averages were given; this happened to some London families with large total expenditures so that the average for London was lower than it should have been.

6.3. The results in Table 1, which are satisfactory on the whole, do not belong to our main analysis and were mainly intended as a preliminary experiment. The use of logarithmic Engel curves and of a multiplicative London effect appears to be successful, but the treatment of children (or rather family composition in general) could be improved. The proportional effect of the addition of a child, as given by (14), is constant, so that the absolute effect will increase with family size. This is not in accordance with the argument in 5.1.4. In the calculations of the Monograph a formula with a unit consumer scale will be used, viz.

$$\log \frac{e_j}{\sum_i k_{ji} n_i} = a_j + b_j \log \frac{e}{\sum_i k_i n_i}, \qquad (15)$$

where the weights of the income scale will be determined by the method of 5.4.4. Some first examples are given in Section 7.

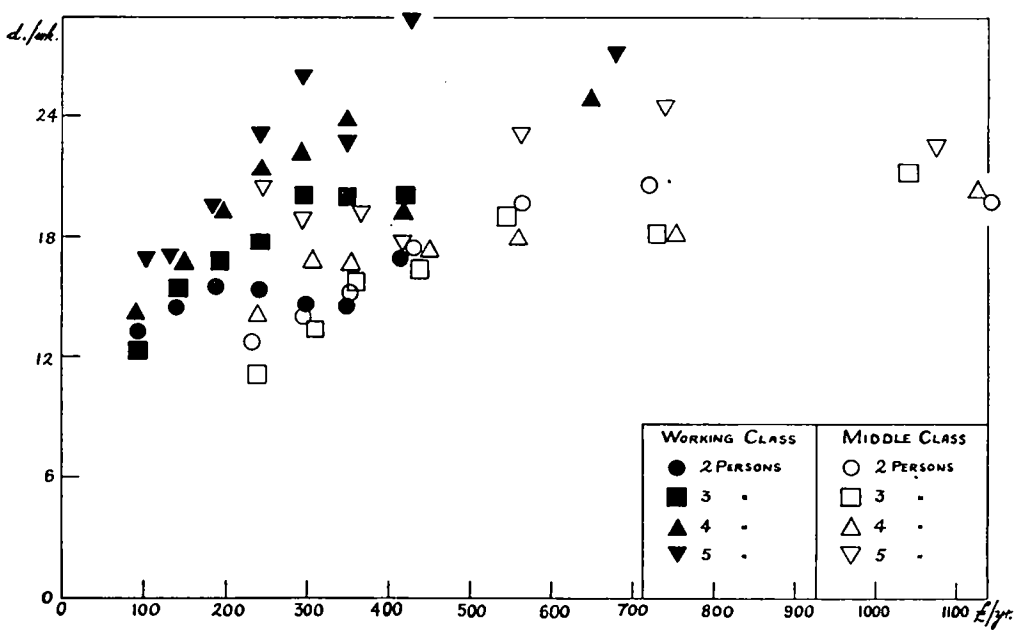

FIG. 6.

7. *Social Class Differences*

7.0. The uniform treatment of middle-class and working-class families in the present analysis makes it possible to detect differences in consumption between these two social classes which cannot be ascribed to total expenditure or family size. The existence of differences of this kind can be established in several cases; Fig. 6 provides a striking example: expenditure on tea is much less in middle-class families than in working-class families of the same total expenditure and number of heads, except perhaps in two-person families. This is probably connected with a higher coffee consumption in middle-class households. As tea and coffee have no nutritional value this difference could hardly be explained by the nature of the work performed (cf. 1.3.3).

7.1. It was consequently necessary to introduce a dummy variable, analogous to the London factor of 6.1, to enable working and middle-class observations to be used in the same regression analysis.* The equation used so far is

$$\log \frac{e_j}{n} = a_j + b_{1j} \log \frac{e}{n} + b_{4j}s, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

with a social class factor s which is 1 for a middle-class household and 0 for a working-class household; this factor is again multiplicative with respect to e_j. The number of heads is used as a family size index. Some estimates are

	b_{1j}	b_{4j}	R^2	*Middle Class per cent Difference.*
Sugar	·33 (±·02)	−·22 (±·03)	·68	−37
Bacon and ham	·64 (±·04)	−·44 (±·05)	·76	−64
Beef and veal (home prod.)	·70 (±·09)	−·08 (±·06)	·65	−17
Pork	·54 (±·05)	−·59 (±·11)	·41	−74
Coffee	1·28 (±·12)	+·85 (±·15)	·82	+610
Cocoa	·25 (±·07)	−·17 (±·09)	·12	−32
Tea	·49 (±·03)	−·40 (±·04)	·70	−60

The social class coefficients are seen to be quite considerable and significant in most cases. In comparing the total expenditure elasticities with those of Table 1 it should be remembered that the Nicholson data cover only a selected sample of 791 working-class households, whereas the above figures also refer to an additional 1,428 working-class and 1,361 middle-class families. The different ways of introducing family size may have affected the total expenditure elasticities, but of the whole the two sets of estimates agree fairly well where they can be compared. When using these elasticities the remark at the end of 4.4 should be borne in mind.

8. *Income and Total Expenditure*

8.0. In 2.2 we have explained the choice of total expenditure as the main explanatory variable, the decisive reason being the absence of income figures in the working-class budgets. For the middle-class families the income of the head was one of the items on the questionnaire. It is not clear how exactly this income was defined, particularly if it included property income, but in view of the paucity of information on the income-consumption relation in Great Britain before the war an attempt to utilize these data has nevertheless been undertaken. While the results cannot be said to be very illuminating for this purpose, they do give rise to some remarks on the design of budget surveys.

8.1. In this calculation only middle-class families with one earner (presumably the head) were considered; they were classified by the income of the head and by the number of persons. The number of observations was 1,095, or about 80 per cent. of the total number of middle-class households in the sample. As a first approach a linear regression was calculated, viz.,

$$e = ·947\ (\pm ·024)M + 10·9\ (\pm 2·5)n + 42·6 \qquad R^2 = ·951. \quad . \quad . \quad (17)$$

* Stuvel and James (1950) think it preferable to analyse the two classes separately, but our results suggest that the social class factor will make this unnecessary and that a uniform income elasticity does not distort the facts unduly.

TABLE 1

No.	*Group (abbreviated)*	*a*	*Total Expenditure Elasticity*	*Children Coefficient*	*London Coefficient*	R^2	*Children % Difference*	*London % Difference*
A1.	Bread	1·34	−0·01±0·06	0·074±0·009	−0·022±0·026	0·69	18·6	−5·0
A2.	Flour	0·50	0·10±0·14	0·074±0·019	−0·191±0·058	0·46	18·6	−35·6
A3.	Cakes, etc.	−1·50	0·92±0·12	0·003±0·017	−0·207±0·051	0·64	·7	−37·9
A4.	Oatmeal, etc.	−2·32	0·92±0·11	0·112±0·015	−0·117±0·046	0·79	29·3	−23·6
A5.	Butcher's meat	0·30	0·46±0·06	−0·011±0·008	0·015±0·025	0·64	−2·6	3·5
A6.	Sausages, etc.	−0·15	0·40±0·08	0·037±0·010	−0·036±0·031	0·54	8·8	−8·0
A7.	Rabbits, etc.	−3·54	1·34±0·35	−0·122±0·048	−0·338±0·144	0·39	−24·5	−54·1
A8.	Bacon, etc.	−0·78	0·70±0·09	0·001±0·013	−0·138±0·038	0·64	·1	−27·3
A9.	Fish	−2·17	1·04±0·02	−0·039±0·017	0·048±0·050	0·70	−8·7	11·6
A10.	Fresh milk, etc.	−0·47	0·66±0·07	0·061±0·009	0·055±0·028	0·80	15·1	13·6
A11.	Condensed milk	2·29	−0·74±0·27	0·142±0·037	0·205±0·113	0·38	38·8	60·4
A12.	Cream	−5·81	1·97±0·24	−0·145±0·033	0·068±0·101	0·72	−28·4	16·9
A13.	Butter	−0·23	0·54±0·08	0·026±0·011	−0·162±0·034	0·65	6·1	−31·2
A14.	Margarine	1·01	−0·23±0·14	0·149±0·019	0·038±0·059	0·63	40·9	9·0
A15.	Lard	−0·39	0·31±0·14	−0·011±0·020	−0·399±0·060	0·56	−2·6	−60·1
A16.	Cheese	−0·44	0·41±0·09	0·031±0·013	0·016±0·039	0·42	7·3	3·7
A17.	Eggs	−0·60	0·63±0·07	0·027±0·010	−0·043±0·030	0·70	6·4	−9·3
A18.	Tea	0·36	0·27±0·05	0·028±0·007	−0·026±0·022	0·54	6·6	−5·8
A19.	Cocoa	−0·86	0·27±0·21	0·142±0·028	−0·126±0·086	0·47	38·7	−25·2
A20.	Coffee	−3·48	1·17±0·31	−0·072±0·043	−0·131±0·131	0·31	−15·2	−26·1
A21.	Sugar	0·36	0·20±0·05	0·058±0·007	−0·098±0·020	0·77	14·3	−20·2
A22.	Jam, etc.	−1·09	0·61±0·12	0·061±0·017	−0·034±0·051	0·52	15·1	−7·6
A23.	Potatoes	0·04	0·29±0·07	0·086±0·009	0·070±0·029	0·74	22·0	17·6
A24.	Vegetables, etc.	−2·11	1·10±0·06	−0·014±0·008	0·055±0·025	0·91	−3·1	13·6
A25.	Fruit	−3·43	1·53±0·11	−0·008±0·015	0·020±0 045	0·85	−1·9	4·7
A26.	(Sweets)	—	—	—	—	—	—	—
A27.	Other foods	−1·80	1·09±0·08	0·011±0·011	−0·078±0·034	0·83	2·7	−16·4
A28.	Meals away from home	−6·05	2·36±0·35	−0·180±0·048	0·388±0·146	0·68	−33·9	144·5
AT.	Total food	0·72	0·60±0·03	0·026±0·005	−0·014±0·014	0·91	6·3	−3·1
BT.	Rent or purchase of dwelling, rates, etc.	0·28	0·62±0·06	−0·019±0·008	0·145±0·024	0·83	−4·3	+39·7
C1.	Clothing, men's	−2·05	1·17±0·10	−0·079±0·014	−0·121±0·043	0·81	−16·6	−24·4
C2.	Clothing, women's	−2·90	1·47±0·10	−0·109±0·014	−0·068±0·042	0·88	−22·3	−14·6
C3.	Clothing, children's	−3·70	1·32±0·47	0·586±0·064	−0·184±0·194	0·73	+285·0	−34·6
C4.	Repairs (clothing)	−4·53	1·58±0·18	−0·082±0·025	0·046±0·075	0·72	−17·3	+11·2
C5.	Boots and shoes	−1·12	0·74±0·06	0·090±0·008	−0·050±0·024	0·90	+22·9	−10·9
C6.	Repairs (boots and shoes)	−1·66	0·82±0·08	0·009±0·011	−0·071±0·033	0·75	+2·0	−15·0
CT.	Total clothing	−1·57	1·16±0·05	0·021±0·007	−0·089±0·022	0·93	+4·9	−18·5

D1.	Coal	−0·15	0·58±0·08	−0·012±0·012	−0·138±0·035	0·61	−2·8	−27·2
D2.	Coke	−1·28	0·36±0·58	−0·150±0·079	−0·161±0·239	0·10	−29·3	−31·0
D3.	Gas	−0·04	0·41±0·07	0·001±0·010	0·232±0·030	0·75	+·2	+71·1
D4.	Electricity	−2·83	1·31±0·21	−0·033±0·028	0·043±0·086	0·55	−7·2	+10·5
D5.	Oil	0·52	−0·18±0·47	−0·010±0·065	−0·310±0·197	0·07	−2·4	−51·0
D6.	Firewood, matches, candles	−0·62	0·49±0·10	−0·005±0·014	−0·133±0·042	0·45	−1·1	−26·5
DT.	Total fuel and light	0·13	0·58±0·04	−0·010±0·006	0·008±0·018	0·84	−2·3	+1·8
E1.	Soap	−0·91	0·62±0·05	0·023±0·007	−0·055±0·020	0·83	+5·5	−12·0
E2.	Soda	−2·09	0·91±0·08	−0·009±0·010	−0·024±0·031	0·80	−2·0	−5·4
E3.	Ironmongery, tools, etc.	−6·33	2·32±0·41	−0·147±0·056	−0·247±0·169	0·52	−28·8	−43·3
E4.	Household brushes, brooms, etc.	−5·60	1·85±0·33	0·034±0·045	−0·403±0·138	0·51	+8·1	−60·4
E5.	Pottery and glassware	−5·02	1·71±0·45	−0·001±0·061	−0·240±0·185	0·30	−·3	−42·5
E6.	Drapery and haberdashery	−5·82	2·22±0·42	−0·077±0·058	−0·222±0·176	0·45	−16·2	−40·0
E7.	Furniture	−6·20	2·42±0·64	0·058±0·088	−0·755±0·266	0·37	+14·2	−82·4
E8.	Carpets	−9·00	3·33±0·54	−0·169±0·074	−0·813±0·223	0·58	−32·3	−84·6
E9.	Other household utensils	−9·55	3·41±0·41	−0·182±0·056	−0·236±0·170	0·69	−34·2	−41·9
E10.	Tobacco and cigarettes	−1·03	0·85±0·07	−0·009±0·010	0·071±0·030	0·81	−2·1	+17·7
E11.	Rail travelling to and from work	−4·93	1·84±0·53	−0·186±0·073	0·491±0·221	0·44	−34·8	+210·0
E12.	Bus travelling to and from work	−4·05	1·68±0·26	−0·047±0·035	0·072±0·106	0·57	−10·3	+18·1
E13.	Other travelling	−5·69	2·24±0·27	−0·087±0·036	−0·014±0·110	0·69	−18·2	−3·2
E14.	Newspapers and periodicals	−0·98	0·70±0·07	−0·030±0·010	−0·032±0·031	0·73	−6·8	−7·1
E15.	Books, stationery, etc.	−4·34	1·59±0·21	−0·067±0·029	0·044±0·087	0·65	−14·3	+10·5
E16.	Postage, telephone, etc.	−3·55	1·46±0·11	−0·100±0·016	−0·045±0·047	0·85	−20·5	−9·8
E17.	Cinemas	−2·95	1·31±0·20	−0·036±0·028	−0·057±0·085	0·55	−7·9	−12·4
E18.	Theatres, dances, etc.	−7·20	2·61±0·30	−0·134±0·041	−0·374±0·125	0·70	−26·5	−57·8
E19.	Sports, etc., admission	−4·38	1·62±0·31	−0·014±0·043	−0·309±0·130	0·45	−3·1	−50·9
E20.	Education, etc.	−8·87	3·01±0·77	0·177±0·106	−0·481±0·321	0·35	+50·2	−66·9
E21.	Hairdressing	−4·34	1·73±0·15	−0·045±0·020	0·059±0·061	0·81	−9·8	+14·6
E22.	Laundry charges	−4·54	1·80±0·22	−0·161±0·031	0·148±0·093	0·74	−30·9	+40·7
E23.	Doctor, dentist, etc.	−7·82	2·95±0·42	0·021±0·058	−0·593±0·174	0·61	+5·0	−75·0
E24.	Medicine, etc.	−3·06	1·29±0·19	−0·018±0·026	−0·140±0·079	0·57	−4·0	−27·6
E25.	Hospital funds	−2·55	1·01±0·23	−0·017±0·031	−0·015±0·094	0·36	−3·8	−3·5
E26.	Unemployment, National Health and Pensions Insurance	0·57	0·23±0·08	−0·002±0·011	0·002±0·034	0·19	−·6	+·5
E27.	Insurance, pension funds, etc.	−3·53	1·69±0·15	−0·019±0·020	−0·092±0·061	0·79	−4·3	−19·2
E28.	Trade Unions, Friendly Societies, etc.	−1·32	0·84±0·24	−0·008±0·032	−0·207±0·098	0·30	−1·8	−38·0
E29.	Licences	−6·92	2·61±0·54	−0·159±0·074	−0·713±0·223	0·48	−30·6	−80·6
E30.	Domestic help	−9·72	3·36±0·65	−0·210±0·089	−0·379±0·269	0·47	−38·4	−58·2
E31.	Holiday expenditure	−9·60	3·52±0·53	−0·256±0·072	−0·610±0·219	0·62	−44·6	−75·4
E32.	Food for animals	−6·04	2·16±0·41	−0·162±0·056	−0·117±0·169	0·50	−31·2	−23·6
E33.	Drink	−6·47	2·52±0·38	−0·102±0·052	−0·213±0·159	0·55	−20·9	−38·8
E34.	Other expenditure	−6·78	2·77±0·15	−0·055±0·020	−0·037±0·061	0·91	−12·0	−8·1
ET.	Total other items	−2·53	1·69±0·03	−0·029±0·004	−0·034±0·012	0·99	−6·5	−7·6

Here M means money income net of income tax and n the number of persons. A logarithmic formula yielded

$$\log e = \cdot 890\ (\pm \cdot 038) \log M + \cdot 345 \qquad R^2 = \cdot 939; \quad . \quad . \quad . \quad (18)$$

the coefficient of the number of persons was not significant. Though satisfactory from the statistical point of view these estimates are rather disturbing because of the low savings figures which they imply ("savings" include insurance premiums in this context, as the latter could not be eliminated). Even comparatively high income families are found to be spending more than they earn. This had in fact already been noticed by Massey (1942, p. 179), who ascribed it to incomes of supplementary earners being left out. In our case this explanation cannot hold, or rather it can hold only for property incomes that were not stated. In principle it would be possible to check on stated incomes by the income-tax payments recorded by the families, but in practice this does not work out very well. However, these tax figures do not suggest that the incomes quoted in the documents were understated.

8.2. It is nevertheless hardly believable that nearly all these families in stable economic situations were dissaving, so that one cannot help doubting the expenditure figures. The main inquiry was held during four weeks spread out over the year, but some families supplied clothing information for the whole year continuously. Mr. Massey (1942, pp. 174–5) discusses the remarkable discrepancy between the 4-weekly and the yearly averages for clothing thus obtained, plausibly explaining it by a tendency to include in the 4-weekly figures some expenditures incurred in other periods. This may well have applied to other items than clothing as well, particularly to those bought infrequently; it does not necessarily bias the elasticity estimates, because the tendency to overestimate clothing estimates was apparently not correlated with income. It might be serious if one tried to estimate national expenditure from budget data, but this is a hazardous procedure in any case. The only way to prevent these discrepancies is to have a continuous inquiry over a longer period, perhaps only for the items where they are most likely to occur.

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The investigations referred to in this paper could not have been undertaken without the generous co-operation of a considerable number of persons and institutions. This was especially true of the Hollerith work, which involved the punching and subsequent manipulation of well over 100,000 cards. The analysis was started as part of an inquiry into the current demand for foodstuffs of which Dr. James Tobin was then in charge; most of the planning was done jointly with him. The Director of the Department of Applied Economics, Mr. J. R. N. Stone, has taken an active interest in all phases of the work. Mr. S. J. Prais has been working on this project from October, 1950, onwards and has made many useful suggestions. The author accepts full responsibility for any remaining defects. The Ministry of Labour and National Service gave access to the transcripts of the returns for the two budget inquiries, and was always ready to help with additional explanations. The Ministries of Food and of Agriculture and Fisheries, the Royal Aircraft Establishment in Farnborough and the College of Aeronautics in Cranfield kindly undertook extensive parts of the Hollerith work and carried them out in the most helpful spirit, despite the many difficulties of this frequently unusual work. The help of the National Physical Laboratory in Teddington and the University Mathematical Laboratory in Cambridge in allowing the use of their Hollerith equipment was also highly valuable; there Mrs. A. E. Gill and Mrs. M. V. Allnutt did an important share of the punched-card work. The large volume of other computing work was carried out by the Department's computing section and at the National Institute of Economic and Social Research. To all who took part the author expresses his sincere gratitude.

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DISCUSSION ON MR. HOUTHAKKER'S PAPER.

Mr. J. L. NICHOLSON: A little while ago we had an admirable paper from Dr. Tobin, who was then also working at Cambridge, on "A Statistical Demand Function for Food in the United States". Mr. Houthakker has now given us a most interesting and illuminating paper which ranges, swiftly but deftly, over a large number of the theoretical problems connected with the analysis of family budgets. The author deserves to be congratulated, not only for the undoubted excellence of his paper, but also for his energy and enterprise in undertaking this work, which must have involved an enormous amount of tabulation and computation alone. We should also, I think, be grateful to Mr. Stone for his patronage of this large-scale undertaking, as well as to the Ministry of Labour for supplying the detailed information and to other Departments for their assistance.

Perhaps I might say a word, first, about the kind of function which is chosen to represent the relationship between income (or in this case total expenditure) and expenditure on particular items. There are some advantages and some disadvantages in using an ordinary polynomial, and the same must be said of logarithmic functions. The additive property of the former is sometimes a very useful safeguard; for instance if one is making a prediction from one level of income to another and the calculation is required in a hurry. A logarithmic function on the other hand may, as Mr. Houthakker suggests, give a better fit at very low incomes, because it takes account automatically of the fact that consumption cannot be negative; and it clearly involves more realistic assumptions about homoscedasticity. I accept the point that although logarithmic functions do not satisfy the additivity condition, the discrepancy in practice is likely to be very small, since the data to which the functions are fitted do of course satisfy this condition. But the same argument applies surely to functions of the ordinary values fitted to data which cannot show negative consumption. However, we cannot really decide which is the best type of function to use on this kind of data until we have had a good deal more experience.

Mr. Houthakker questions whether the average expenditure on clothing, shown in the four weeks of the Ministry of Labour's inquiry, should be adjusted and brought into line with the annual data on clothing for the same households. Expenditure on certain items, such as clothing, sometimes has a habit of being recorded if it occurs just before or just after the week of the inquiry. This would tend to raise the figures of total expenditure, but not perhaps by as much as the expenditure on clothing (and similar items) is raised, because of the tendency for errors to compensate. Whether one is likely to get nearer the truth by adjusting the figures shown in the main part of the inquiry or by leaving them unaltered is a very nice point.